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CONTROLLING THE MOVEMENT OF HEXACOPTER ALONG THE INTENDED ROUTE WITH ENGINE FAILURE

Summary. This article investigates the issue of controlling the movement of a hexacopter-type unmanned aerial vehicle around a route. The movement of the hexacopter is modeled as the motion of a rigid body, taking into account gravitational forces and aerodynamic resistance forces. The spatial orientation of the hexacopter is expressed using quaternions. The movement route is considered as a broken line consisting of straight-line segments, and parameters that control the hexacopter's flight on the considered straight-line segment of the route are determined when one of its engines fails. The mathematical rationale for how to control the operational engines to continue the hexacopter's movement as before in the event of an engine failure is provided.
Keywords: hexacopter, route, control parameters, engine failure, quaternion, spatial orientation, unmanned aerial vehicle

1. INTRODUCTION

Recently, with the widespread application of multi-engine drones, various types have become particularly popular depending on their purpose and the demands placed upon them [1, 2, 3]. Unlike single-engine drones, the failure of engines in multi-rotor devices can lead to safety-related issues. Numerous published articles suggest solving this problem by redesigning

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the control law or control power [4, 5]. However, this approach is challenging to implement because altering the control power in this manner usually requires the addition of extra devices.

Several studies have been conducted on the detection of engine failure, maneuverability of unmanned aerial vehicles (UAVs) with engine failure, and the distribution of control among engines [6-8]. Rapid detection of an engine failure in a UAV is crucial for subsequent actions. A novel method for fault diagnosis using thermal imaging was presented in [9]. Lu [10] proposed a Fault Detection and Isolation (FDI) system capable of instantly detecting and isolating a failed engine in quadcopters with a completely failed engine. Merheb [11] suggested a search table to convert a quadcopter into a tri-rotor in the event of a complete failure of one rotor. Nagarjuna and Suresh developed a safe landing sequence [12]. Lee and colleagues used two servo motors to control the relative roll and pitch attitudes of a quadcopter to maintain its stability when a motor failure is detected [13]. Wang and Zhang restructured control commands using a sliding mode control algorithm [14].

Regarding the above-mentioned studies, several questions remain unanswered: first, most previous research has only considered quadcopters with a single engine failure. Secondly, there is not enough material on how to control a hexacopter using quaternion theory methods in such situations.

In this article, the issue of controlling a hexacopter when one of its engines fails under power constraints is examined. The proposed system can assist in the control design when there is an engine failure and increase the likelihood of a successful emergency landing.

For clarity, the engines of the hexacopter will be numbered in the sequence shown in Figure 1. The rotational directions of the engine propellers are schematically presented in the same figure.

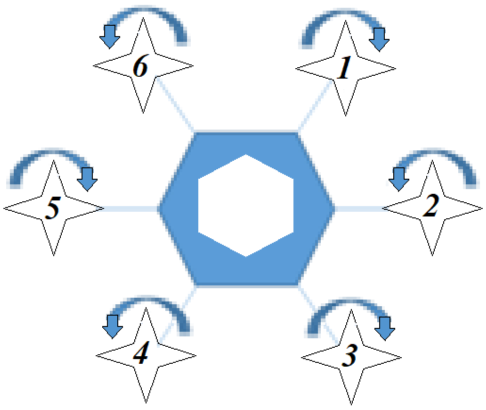

Fig. 1. Rotation direction of the hexacopter’s propellers

In scientific and technical literature, various simulation models of hexacopter movement can be found. Depending on the characteristics of the sensors used in solving the feedback automatic control problem, the flight models of hexacopters differ from one another. During the research, the use of MPU6050 sensors in the studied UAV made it more appropriate to use quaternions as orientation parameters in its mathematical model [15]. This is because MPU6050 sensors measure the rate of change of orientation angles rather than the angles themselves. Numerous articles by various authors have been dedicated to the model of a UAV expressed using quaternions [16, 17, 18, 19]. The model considered here essentially corresponds to the model in [17].

2. PROBLEM STATEMENT

When considering the case where there are no power limitations on the hexacopter's engines (referred to as the normal case below), it becomes apparent that the control of a hexacopter along a straight-line trajectory, even with one engine failure, is fundamentally similar to the control of a quadcopter. This situation also suggests that a hexacopter can be effectively controlled along a straight-line trajectory even if two symmetrically positioned engines fail. The failure of one engine in a hexacopter refers to the scenario where one of its six engines is non-operational. In such cases, it is typically recommended to shut down the engine symmetrically positioned with respect to the hexacopter's center. It is evident that when the number of engines is reduced from six to four, their power needs to be increased. However, a question arises: can the operation of one failed engine be compensated by the remaining five engines in a hexacopter if there are power limitations? This article investigates this issue. Below, the mathematical formalization and solution of the problem are provided.

The mathematical model of the hexacopter is expressed through the interaction between quantities calculated in local and global coordinate systems. Let us introduce the coordinate systems used, as shown in the following figure (Figure 2):

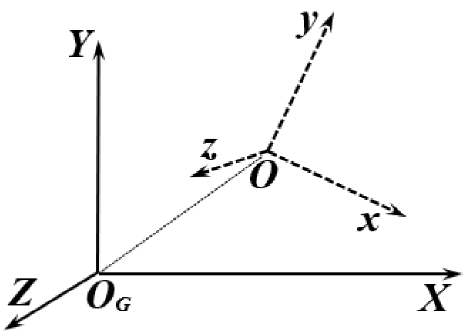

Fig. 2. The local and inertial coordinate systems

O_GXYZ is the inertial coordinate system associated with the ground, while oxyz is the local coordinate system linked to the hexacopter, with its origin at the hexacopter's center of gravity, used to determine its orientation in space.

For clarity, let's assume that the origin O_G of the O_GXYZ system is fixed at a certain point on the Earth's surface. The O_GY axis of the O_GXYZ coordinate system points north, the O_GX axis points east, and the O_GZ axis is directed upwards, perpendicular to both the O_GX and O_GY axes.

Let's assume that the ox axis of the $Oxyz$ system is aligned along the hexacopter's first arm, the Oy axis is perpendicular to the ox axis and lies in the plane of the hexacopter's arms, and the Oz axis is aligned along the hexacopter's symmetry axis, perpendicular to the Oxy plane. In the case of horizontal stillness, it is assumed that the oz axis is directed upwards, and the $Oxyz$ system is a right-handed coordinate system.

3. ORIENTATION GIVEN BY QUATERNIONS

Brief information about quaternions is needed to express the orientation of the UAV in this study.

The general form of a quaternion is given as $q = q_0 + q_1 i + q_2 j + q_3 k$, which is a 4-dimensional hypercomplex number [15]. Here, q_0, q_1, q_2, q_3 are real numbers, and i, j, k are imaginary units. The multidimensional nature of quaternions makes them a convenient and adequate tool for representing rotation angles. Using quaternions allows us to represent the spatial position of a flying vehicle as follows:

$$q = \begin{pmatrix} \cos\varphi \\ u_1 \sin\dfrac{\varphi}{2} \\ u_2 \sin\dfrac{\varphi}{2} \\ u_3 \sin\dfrac{\varphi}{2} \end{pmatrix} \tag{1}$$

Here $u = (u_1, u_2, u_3)$ is the rotation axis vector to achieve the current rotational state of the aircraft, and φ is the principal rotation angle [20]. Specifically, if the rotation axis coincides with the oy axis, and φ is the principal rotation angle, then $u = (0,1,0)$ and:

$$q = \begin{pmatrix} \cos\dfrac{\varphi}{2} \\ 0 \\ \sin\dfrac{\varphi}{2} \\ 0 \end{pmatrix} \tag{2}$$

Assume that the local coordinate system oxyz is rotated by a vector q relative to the inertial coordinate system $O_G XYZ$. Then, to calculate the coordinates of a vector given in the local coordinate system in the inertial coordinate system, the following transformation matrix can be applied [20]:

$$Q = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1 q_2 + q_0 q_3) & 2(q_1 q_3 - q_0 q_2) \\ 2(q_1 q_2 - q_0 q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_3 q_2 + q_0 q_1) \\ 2(q_1 q_3 + q_0 q_2) & 2(q_1 q_3 + q_0 q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} \tag{3}$$

It is clear that to find the coordinates of a vector given in the $O_G XYZ$coordinate system relative to the oxyz coordinate system, the inverse of the transformation matrix Q^{-1} should be applied. The elements of the inverse matrix are denoted as follows:

$$Q^{-1} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}. \tag{4}$$

4. MATHEMATICAL MODEL OF HEXACOPTER

As mentioned earlier, for simplicity, the mathematical model of the UAV will not consider gravitational and aerodynamic forces.

At a given time $t \geq 0$, let the hexacopter's center of gravity have the coordinates $O_G XYZ$ in the $X(t), Y(t), Z(t)$ inertial coordinate system. Also, let the orientation of the local $oxyz$ coordinate system relative to the $O_G XYZ$ inertial coordinate system be expressed by the quaternion $q_0(t) + q_1(t)i + q_2(t)j + q_3(t)k$. Denote the angular velocity of the rotation of the i-th rotor of the hexacopter by ω_i. Then, the movement of the hexacopter, i.e., the rotation of the rotors $\omega = (\omega_1, ..., \omega_6)$ and the translational movement $x(t) = (x(t), y(t), z(t))$, will be described by the following equations, where the rotation vector and the quaternion $q = (q_0(t), q_1(t), q_2(t), q_3(t))$ are interrelated. If we denote the velocity of the hexacopter in the local $oxyz$coordinate system as $v(t) = (v_x(t), v_y(t), v_z(t))$, then the equations of motion expressed in quaternions for the hexacopter will be written as follows [21]:

$$m \frac{dv(t)}{dt} = -c_A g p - c_A |v(t)| v(t) + |w|^2. \tag{5}$$

Here, m is the mass of the hexacopter, c_A is the drag coefficient, g is the acceleration due to gravity, and p is a vector composed of the elements of the last column of the inverse matrix Q^{-1}.

If we denote the angular velocity of the hexacopter as $w(t) = (w_x(t), w_y(t), w_z(t))$, then, based on the total moment $M = (M_1, M_2, M_3)$ exerted on it, the following equations can be written [19, 20]:

$$J \frac{dw(t)}{dt} + w \times Jw = M. \tag{6}$$

Here J is the inertia matrix. The moment M depends on the velocities $\omega_1, \omega_2 ..., \omega_6$ as follows [17]:

$$M = \begin{bmatrix} \frac{\sqrt{3}}{2} k l_0 \cos\alpha(-\omega_2^2 + \omega_3^2 + \omega_5^2 - \omega_6^2) \\ k l_0 \cos\alpha(\omega_1^2 - \frac{1}{2}\omega_2^2 + \omega_3^2 - \omega_4^2 + \frac{1}{2}\omega_5^2 - \omega_6^2) \\ b(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2 + \omega_5^2 - \omega_6^2) \end{bmatrix}. \tag{7}$$

Here, l_0 is the distance between the center of gravity of the hexacopter's main body and the center of gravity of the motor located at the end of its arm (Figure 3), k is the thrust coefficient of the motor, and b is the drag coefficient.

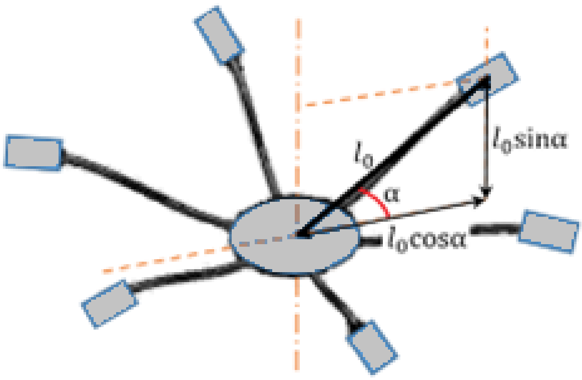

Fig. 3. Structural dimensions of the hexacopter

It should be noted that starting from the angular velocity w found in equation (6), the current orientation quaternion of the hexacopter can be calculated by solving the following system of ordinary differential equations (Poisson's kinematic equations) [21]:

$$\dot{q} = \frac{1}{2} q \otimes w \quad . \tag{8}$$

5. MATHEMATICAL FORMALIZATION OF PROBLEM

As mentioned above, for simplicity, the mathematical model of the UAV takes into account gravitational force, aerodynamic forces, and the thrust force of the engines. The third equation, which directly includes the rotational speeds of the engines, can be written as follows:

$$\sum_{i=1}^{6} \omega_i^2 = f_0 \, . \tag{9}$$

Here:

$$f_0 = c_A |v| v_z + mg \cos \varphi . \tag{10}$$

Considering the condition that the torques generated by the hexacopter's engines are equal to zero during its linear motion, in equation (7):

$$\begin{cases} -\omega_2^2 + \omega_3^2 + \omega_5^2 - \omega_6^2 = 0, \\ \omega_1^2 - \frac{1}{2}\omega_2^2 + \omega_3^2 - \omega_4^2 + \frac{1}{2}\omega_5^2 - \omega_6^2 = 0, \\ \omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2 + \omega_5^2 - \omega_6^2 = 0. \end{cases} \tag{11}$$

Thus, in order to provide the linear motion of the hexacopter at a certain velocity v, the quantities $\omega_1, \omega_2 ..., \omega_6$ must satisfy the system of equations (9)-(11).

6. PROVIDING CONTROL IN NORMAL OPERATING MODE.

Assume that all the engines of hexacopter are functioning normally. In this case, let us investigate the problem of determining the quantities $\omega_1, \omega_2 ..., \omega_6$ that satisfy the system of equations (9)-(11).

For every k, let us denote ω_k^2 as ξ_k. Then the system (9)-(11) can be written as follows:

$$\begin{cases} -\xi_2 + \xi_3 + 2\xi_5 - \xi_6 = 0, \\ 2\xi_1 - \xi_2 + 2\xi_3 - 2\xi_4 + \xi_5 - \xi_6 = 0, \\ \xi_1 - \xi_2 + \xi_3 - \xi_4 + \xi_5 - \xi_6 = 0, \\ \xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 + \xi_6 = 0. \end{cases} \quad (12)$$

As it can be seen, equation (12) is a system of linear equations written with respect to six unknowns, and its rank is equal to four. Therefore, this system has infinitely many distinct solutions. The following optimality criterion can be used to select the most suitable solution from the set of possible solutions that align with the essence of the problem:

$$\Im = \sum_{i \neq j} (\xi_i - \xi_j)^2 \to \min, \; i, j = 1,...,6 . \quad (13)$$

The minimization of the $\Im$ functional essentially requires that the quantities ξ_i, and ultimately the rotation frequencies ω_k^2, be as close as possible to each other. This requirement is justified by the fact that during the control of the straight-line motion of the UAV, its engines should be loaded as equally as possible.

Mathematically, the problem defined by equations (12) and (13) is a constrained optimization problem with respect to the variables ξ_k. To solve this, the Lagrange multipliers method can be used [20]. For this purpose, let us denote the expressions on the left side of the equations (12) as $\mu_1, \mu_2, \mu_3, \mu_4$. Then, by introducing the multipliers $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, the Lagrange function for the problem defined by equations (12) and (13) can be written as follows:

$$\Lambda = \Im + \lambda_1\mu_1 + \lambda_2\mu_2 + \lambda_3\mu_3 + \lambda_4\mu_4 . \quad (14)$$

Thus, the constrained optimization problem defined by equations (12) and (13) reduces to the problem of finding the unconstrained minimum of the functional (14). To find the minimum of the Λ functional, let's compute its partial derivatives with respect to the variables $\xi_1,...,\xi_6$ and $\lambda_1,...,\lambda_4$ and set them equal to zero. This will result in the following system of equations:

$$
\begin{cases}
10\xi_1 - 2\xi_2 - 2\xi_3 - 2\xi_4 - 2\xi_5 - 2\xi_6 + \lambda_2 + \lambda_3 + \lambda_4 = 0, \\
-4\xi_1 + 20\xi_2 - 4\xi_3 - 4\xi_4 - 4\xi_5 - 4\xi_6 - 2\lambda_1 - \lambda_2 - 2\lambda_3 + 2\lambda_4 = 0, \\
-2\xi_1 - 2\xi_2 + 10\xi_3 - 2\xi_4 - 2\xi_5 - 2\xi_6 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0, \\
-2\xi_1 - 2\xi_2 - 2\xi_3 + 10\xi_4 - 2\xi_5 - 2\xi_6 - \lambda_2 - \lambda_3 + \lambda_4 = 0, \\
-4\xi_1 - 4\xi_2 - 4\xi_3 - 4\xi_4 + 20\xi_5 - 4\xi_6 + 2\lambda_1 + \lambda_2 + 2\lambda_3 + 2\lambda_4 = 0, \\
-2\xi_1 - 2\xi_2 - 2\xi_3 - 2\xi_4 - 2\xi_5 + 10\xi_6 - \lambda_1 - \lambda_2 - \lambda_3 + \lambda_4 = 0, \\
-\xi_2 + \xi_3 + 2\xi_5 - \xi_6 = 0, \\
2\xi_1 - \xi_2 + 2\xi_3 - 2\xi_4 + \xi_5 - 2\xi_6 = 0, \\
\xi_1 - \xi_2 + \xi_3 - \xi_4 + \xi_5 - \xi_6 = 0, \\
\xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 + \xi_6 = 0.
\end{cases} \tag{15}
$$

If we solve this system of equations using Cramer's rule, we will obtain the following results:

$$\xi_1 = \xi_2 = \xi_3 = \xi_4 = \xi_5 = \xi_6 \approx 0{,}166\, f_0 ,\ (\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0)\,. \tag{16}$$

Based on the calculated values of the quantities $\xi_1,\ldots,\xi_6$, we obtain the following values for the rotation frequencies of the propellers:

$$\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega_5 = \omega_6 \approx 0{,}4\sqrt{c_A |v| v_z + mg\cos\varphi}\,. \tag{17}$$

Thus, for the hexacopter to fly in a straight line, it is first brought into the appropriate orientation and achieves the desired pitch by adjusting the rotation frequencies of the propellers. After that, it is controlled along the corresponding trajectory using the engines operating at the rotation frequencies given by equation (17). (It should be noted that the calculation of the propeller rotation frequencies required for changing the UAV's orientation is not considered in this paper).

7. CONTROLLING THE HEXACOPTER WITH ENGINE FAILURE

As seen from (17), the optimal control of straight-line flight when all motors are operating normally is provided by rotating all propellers at the same frequency. Suppose one of the hexacopter's motors has failed. Without generalizing, it can be assumed that the failed motor is, for example, the 6th motor. In the absence of constraints on the rotation frequencies of the motors, the issue of controlling the hexacopter's movement has been addressed in [21], and it has been shown that control is possible along a straight-line trajectory when $\omega_3 = 0$. As mentioned above, a question arises: if the power of the motors is insufficient, and they cannot achieve the rotation frequencies given in (17), can the hexacopter be controlled in the previous mode using 5 motors?

The failure of the 6^{th} motor means that $\omega_6 = 0$ must be taken when solving the system (9)-(11). Thus, the system (9)-(11) is transformed into a system of four equations written with respect to five unknowns. After substituting $\xi_k = \omega_k^2,\ k = (1,2,\ldots,5)$ the analog of system (17) is written as follows:

$$\begin{cases} -\xi_2 + \xi_3 + 2\xi_5 = 0, \\ 2\xi_1 - \xi_2 + 2\xi_3 - 2\xi_4 + \xi_5 = 0, \\ \xi_1 - \xi_2 + \xi_3 - \xi_4 + \xi_5 = 0, \\ \xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 = 0. \end{cases} \tag{18}$$

To choose the most suitable solution from the set of possible solutions according to the essence of the problem, we can take the following optimality criterion as the analog of functional (13):

$$\Im = \sum_{i \neq j} (\xi_i - \xi_j)^2 \to \min,\ i, j = 1,...,5. \tag{19}$$

In this case, let's solve the problem with constraints applied to equations (18). If the 6th engine is not working, then the problem with constraints will be as follows:

$$\begin{cases} -\xi_2 + \xi_3 + 2\xi_5 = 0, \\ 2\xi_1 - \xi_2 + 2\xi_3 - 2\xi_4 + \xi_5 = 0, \\ \xi_1 - \xi_2 + \xi_3 - \xi_4 + \xi_5 = 0, \\ \xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 = 0, \\ \xi_1 - \xi_0 \le 0, \\ \xi_2 - \xi_0 \le 0, \\ \xi_3 - \xi_0 \le 0, \\ \xi_4 - \xi_0 \le 0, \\ \xi_5 - \xi_0 \le 0. \end{cases} \tag{20}$$

This problem is mathematically a conditional extremum problem. Various approaches can be applied to solve the problem [4]. During the research, the Kuhn-Tucker method was used [22]. If a solution to this problem exists, then it must satisfy all the minima obtained by solving with each of the individual additional conditions:

$$\begin{cases} 8\xi_1 - 2\xi_2 - 2\xi_3 - 2\xi_4 - 2\xi_5 + \lambda_2 + \lambda_3 + \lambda_4 = 0, \\ -4\xi_1 + 16\xi_2 - 4\xi_3 - 4\xi_4 - 4\xi_5 - 2\lambda_1 - \lambda_2 - 2\lambda_3 + 2\lambda_4 = 0, \\ -2\xi_1 - 2\xi_2 + 8\xi_3 - 2\xi_4 - 2\xi_5 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0, \\ -2\xi_1 - 2\xi_2 - 2\xi_3 + 8\xi_4 - 2\xi_5 - \lambda_2 - \lambda_3 + \lambda_4 = 0, \\ -4\xi_1 - 4\xi_2 - 4\xi_3 - 4\xi_4 + 16\xi_5 + 2\lambda_1 + \lambda_2 + 2\lambda_3 + 2\lambda_4 = 0, \\ -2\xi_1 - 2\xi_2 - 2\xi_3 - 2\xi_4 - 2\xi_5 - \lambda_1 - \lambda_2 - \lambda_3 + \lambda_4 = 0, \\ -\xi_2 + \xi_3 + 2\xi_5 = 0, \\ 2\xi_1 - \xi_2 + 2\xi_3 - 2\xi_4 + \xi_5 = 0, \\ \xi_1 - \xi_2 + \xi_3 - \xi_4 + \xi_5 = 0, \\ \xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 = 0, \\ \xi_k - \xi_0 \le 0, \\ k = 1,2,...,5. \end{cases} \tag{21}$$

For $k \neq 2$, this system of equations can be solved with each of the corresponding constraint conditions, and it turns out that the system is consistent. However, when the system is solved with the second constraint condition, it becomes apparent that there is no solution to this problem. This means that when constraints are applied to the engines, it is not possible to compensate for the failure of one engine using the remaining five engines.

It is clear that the same result is obtained if any of the 2^{nd}, 3^{rd}, or 5^{th} engines fail.

It should be noted that the terms related to the 1st and 5th engines are not included in the 1st equation of system $(18) - (\xi_2 + \xi_3 + \xi_5)$. This is due to the placement of these engines relative to the $oxyz$ coordinate system. For clarity, if we consider the case where the 1st engine fails instead of the 6^{th} engine, by solving the resulting system in a similar manner, we again conclude that the hexacopter cannot be controlled with five engines under the given power constraints. Naturally, the results are similar when the 4^{th} engine fails.

Thus, this means that under the given constraints, it is not possible to control the hexacopter along a straight-line trajectory using only five engines.

8. CONCLUSIONS

Thus, the research showed that when one of the hexacopter's engines fails, the continuation of its movement along the previous trajectory can be ensured by the other engines, except for the engine symmetrically positioned relative to the failed one. In this case, if there are no technical limitations on the power of the engines, it is necessary to increase the rotation frequency of the propellers to continue the movement at the previous speed.

However, if there are power limitations on the engines, the continuation of the flight along the trajectory can be achieved by reducing the movement speed. It was also mathematically substantiated that under such limitations, the power deficiency across four engines cannot be compensated by the fifth engine to maintain the hexacopter's flight speed along a straight trajectory at its previous level.

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