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ACCURACY ANALYSIS OF AIRCRAFT POSITIONING USING REAL RADAR AND GPS DATA

Summary. This paper presents an analysis of the accuracy of aircraft positioning using radar and GPS satellite data. In particular, this study shows the results of research on determining the position of an aircraft, as well as the range and azimuth parameters for the GCA-2000 radar to the GPS solution. The research used measurement data from the GCA-2000 radar and the Thales MobileMapper Pro receiver placed onboard a Diamond DA-40NG aircraft. The flight experiment was carried out at the EPDE military airport in Dęblin. It was found that the average error in determining the position of the aircraft for the GCA-2000 radar was 295.57 m. Moreover, the average error in determining the range for the GCA-2000 radar is 138.12 m. Additionally, the average error in determining the azimuth for the GCA-2000 radar is equal to 0.408°.

Keywords: radar, GCA-2000, GPS, polar coordinates, Cartesian coordinates, accuracy

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1. INTRODUCTION

Since the early days of aviation, one of the main issues has been to detect, track and determine as accurately as possible the position of aircraft in space. A breakthrough in this field came with the introduction of the first radars by Great Britain in the late 1930s. Their main task was to detect and determine the distance to the attack groups of the German Luftwaffe. Their effectiveness was one of the key reasons for the victory of the Allied forces in the Battle of Britain. This was followed by the rapid development of radar technology. After the war, radar technology found application in the field of air traffic control, both military and civilian [1, 2].

Radar is a device whose operation is based on the use of electromagnetic waves. By transmitting an appropriate radio wave and receiving its echo reflected from an object, it detects and determines its position in space in real-time. The use of radar systems allows continuous observation 24 hours a day. The way radar works make it possible to determine the distance, bearing and even the speed of an object [3].

With the increase in air traffic operations, radar has become an essential tool used by air traffic control services, whose mission is to ensure an orderly, efficient and safe flow of air traffic and prevent aircraft collisions both on the ground and in the air. Air traffic control radars enable the observation of large volumes of airspace and the tracking of very large numbers of aircraft in real-time. The accuracy of radar positioning is essential to ensure adequate separation and guidance of aircraft by air controllers.

2. SCIENTIFIC KNOWLEDGE ANALYSIS

In Poland, navigation radars have been used in several scientific and research works. The research was carried out given the usefulness of navigation radars in positioning aircraft with GNSS satellite measurements. In work [4], the positioning accuracy of the PZL-130 Orlik aircraft was determined during a test flight around the Radom airport. In the study, a comparison was made between the readings of horizontal and vertical coordinates of the aircraft determined by radar and an onboard GPS receiver. The paper [5] analysed the application of the RST-12M radar in the precise positioning of the Cessna aircraft. The following parameters were analysed: accuracy of determining the radar-aircraft slant range parameter, accuracy of determining the azimuth parameter, and accuracy of determining the flight altitude parameter. The study [6] analysed the application of the NUR-22N-(3D) radar in the precise positioning of two PZL-130 Orlik aircraft. The following analyses were carried out: radar distinguishability as a function of range and radar distinguishability as a function of azimuth. In the paper [7], a study was carried out to determine the accuracy of the radiolocation station during the flight test. In the research, the radar-aircraft slant range, azimuth, and flight altitude for a TS 11 Iskra aircraft were determined.

Based on the analysis of the state of the art, it can be concluded that:

- research topics related to the use of radar in air navigation are very important,
- in Poland, research on the use of radar in air navigation has been conducted by the Air Force Institute of Technology, the Military Institute of Armament Technology and the Polish Air Force University,
- the presented research [4-7], mainly investigated the accuracy of aircraft positioning using radar data,

- further work is needed on this research topic to validate and verify the real-time performance of the radar.

The main objective of this work is to determine the accuracy of the coordinates determinationof an aircraft by the GCA-2000 radar, used by the approach control (APP) service at the airport in Dęblin. In the flight experiment, the coordinates of the aircraft in flight were recorded by the GCA-2000 radar and the Thales MobileMapper Pro GPS receiver. To determine the accuracy of positioning the aircraft by radar, a comparison was made between navigation readings from the GCA-2000 radar and the GPS receiver. The result of the performed test and numerical analysis is the identification of accuracy of the aircraft position determination by radar.

This article consists of 6 sections: 1 - Introduction, 2 - Analysis of the state of knowledge, 3 - Research method, 4 - Research test, 5 - Test results and discussion, and 6 - Conclusions. The bibliography is presented at the end of the work.

3. RESEARCH METHOD

Accuracy is considered to be the degree of comparison of the coordinate values determined by the GCA-2000 radar against the corresponding GPS coordinates for each radar measurement moment [8]. The coordinates from the GPS receiver are considered as the flight reference position. To determine the quality of the positions recorded by the radar with the data recorded by the GPS receiver, calculations were performed for coordinate pairs in Cartesian and polar systems. The calculated measures of accuracy are absolute error, mean absolute error, standard deviation, and RMS mean square error.

In the Cartesian system, the distance between the reference point and the corresponding point determined by the radar is the absolute position error calculated from the formula 1:

$$d = \sqrt{(X_{GCA} - X_{GPS})^2 + (Y_{GCA} - Y_{GPS})^2} \tag{1}$$

where:
(X_{GCA}, Y_{GCA})- Cartesian coordinates of the aircraft based on radar data,
(X_{GPS}, Y_{GPS})- Cartesian coordinates of the aircraft from the GPS solution.

The mean absolute position errors for the whole flight were then calculated using formula 2, as shown below:

$$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n} \tag{2}$$

where:
i- a single measurement,
n- total number of measurements.

The mean absolute error indicates how much the measured value of a parameter differs from the true value.

The next step was to calculate the standard deviation of the position error (σ_d) and the mean square error of position ($RMSE_P$) calculated with the formulas 3 and 4, respectively, as shown below:

$$\sigma_d = \sqrt{\frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n}} \tag{3}$$

$$RMSE_P = \sqrt{\frac{\sum_{i=1}^{n} d_i^2}{n}} \tag{4}$$

The standard deviation of the position error reports how much the individual absolute position error values differ from their averaged value, while the mean squared error reports how the measured values differ from the real values.

The accuracy of the horizontal range and azimuth was determined in the radar polar coordinate system. For this purpose, the respective values of azimuth (Δβ) and range (ΔR) errors, mean absolute errors of azimuth ($\Delta\bar{\beta}$) and range ($\overline{\Delta R}$), the standard deviation of the azimuth error (σ_β) and range error (σ_R) and the mean square error of azimuth ($RMSE_\beta$) and range ($RMSE_R$), were calculated respectively with the formulas 5, 6, 7, 8, 9, 10, 11, and 12, as shown below:

$$\Delta\beta = |\beta_{GPS} - \beta_{GCA}| \tag{5}$$

$$\Delta R = |R_{GPS} - R_{GCA}| \tag{6}$$

$$\Delta\bar{\beta} = \frac{\sum_{i=1}^{n} \Delta\beta_i}{n} \tag{7}$$

$$\overline{\Delta R} = \frac{\sum_{i=1}^{n} \Delta R_i}{n} \tag{8}$$

$$\sigma_\beta = \sqrt{\frac{\sum_{i=1}^{n}\left(\Delta\beta_i - \Delta\bar{\beta}\right)^2}{n}} \tag{9}$$

$$\sigma_R = \sqrt{\frac{\sum_{i=1}^{n}(\Delta R_i - \Delta\bar{R})^2}{n}} \tag{10}$$

$$RMSE_\beta = \sqrt{\frac{\sum_{i=1}^{n} \Delta\beta_i^2}{n}} \tag{11}$$

$$RMSE_R = \sqrt{\frac{\sum_{i=1}^{n} \Delta R_i^2}{n}} \tag{12}$$

where:
(β_{GCA}, R_{GCA})- polar coordinates of the aircraft based on radar data,
(β_{GPS}, R_{GPS})- polar coordinates of the aircraft from the GPS solution.

The described mathematical algorithm (1-12) will be tested and verified in a flight experiment.

4. RESEARCH TEST

To calculate the accuracy of the coordinates determined by the GCA-2000 radar [9], it was necessary to acquire the data recorded by the radar, as well as the reference ("true") position of the aircraft. Geodetic coordinates recorded by the GPS receiver were used as reference data. This study was based on the performance of a flight logged by a GCA-2000 radar located at the airport in Dęblin. The test flight was carried out in cooperation with the Academic Air Training Centre of the Polish Air Force University in Dęblin. The Thales MobileMapper Pro GPS receiver [10] was carried onboard the Diamond DA-40NG aircraft with registration SP-MKL (Figure 1) owned by the Academic Air Training Centre of the Polish Air Force University in Dęblin.

Fig. 1. Picture example of Diamond DA-40NG aircraft [11]

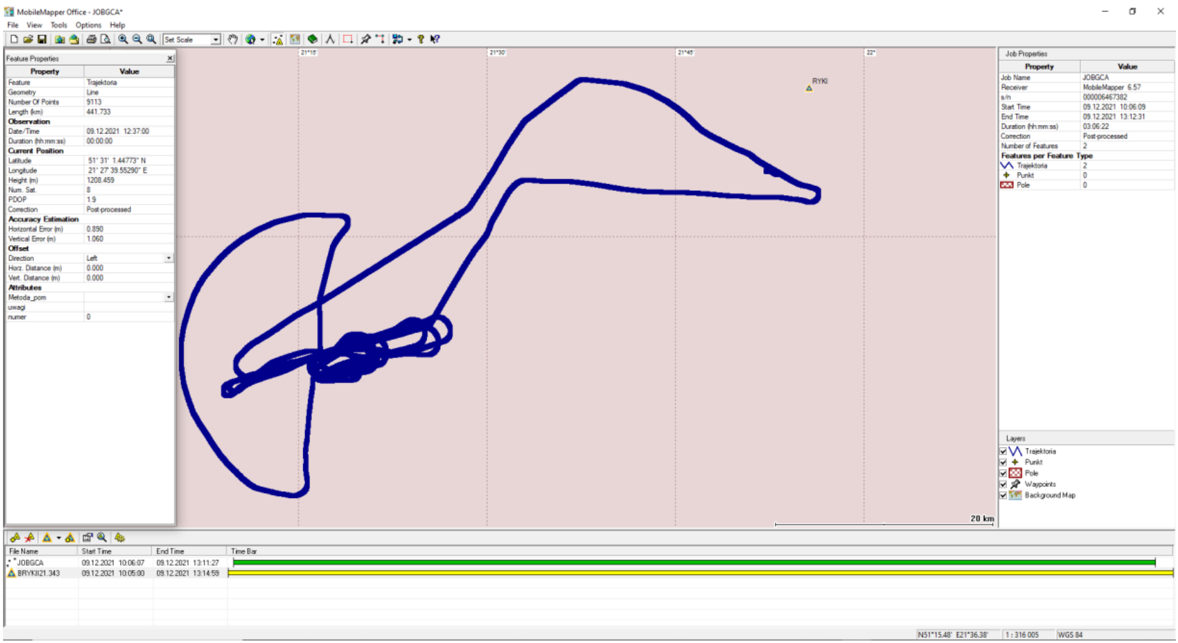

Fig. 2. The horizontal trajectory of the Diamond DA-40NG aircraft during the test in the MobileMapper Office software

The test flight commenced at 09:28:51 UTC on 9 December 2021 from EPDE airport (Figure 2). The flight duration was 2 hours, 22 minutes and 2 seconds and ended at EPDE airport at 11:50:53 UTC. During the flight, the aircraft covered a distance of approximately 440 km at an average speed of approximately 190 km/h. A GPS receiver recorded the position data of the aircraft every second throughout the flight. At the same time, the polar coordinates of successive aircraft detection points were recorded on the ground by the GCA-2000 radar. After the test flight, the measurement epochs were synchronised and navigation calculations were performed using the radar and GPS data. Navigational calculations were performed using the software: MobileMapper Office [12], TRANSPOL v.2.06 [13], and finally the scripts in Scilab v.6.0.0 [14]. The obtained results of the navigation tests are shown in section 5.

5. RESEARCH RESULTS AND DISCUSSION

The analysis of the obtained test results began by presenting the value of the absolute position error according to formula 1. Figure 3 shows the results of the absolute position error of the aircraft as a function of the distance from the radar.

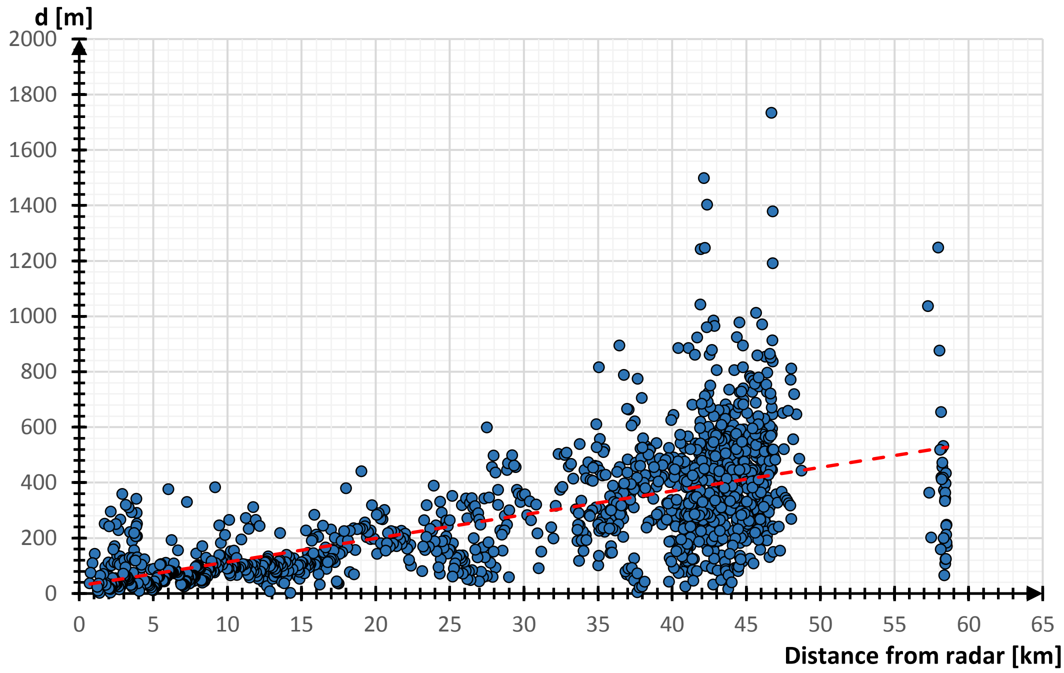

Fig. 3 The absolute error of position of the aircraft as a function of distance from the radar

Absolute position error d is the distance between the corresponding points recorded by the GCA-2000 radar and the GPS satellite receiver. It can be seen from Figure 3 that the value of the position error increases as the distance of the aircraft from the radar increases. The determining influence on the average value of the absolute position error $\bar{d}$ is the accumulation of points in the range of distances from 40 to 48 km. It can be seen that the values of the absolute errors of the aircraft position at distances less than 35 km from the radar in most of the analysed cases are below the average value $\bar{d}$. Moreover, it was noted that the manoeuvring of the aircraft significantly influences the magnitude of the absolute position error. The absolute position error d reaches high values at the moments when the aircraft was turning. At times when the flight was performed in a straight line, the absolute position error values d are significantly lower. This is the reason why in the range of 40-48 km the error values are so high. In this distance range, the aircraft made a large number of holding turns in the vicinity of the EPRA airport. The maximum absolute position error d of approximately 1734 m falls on the moment of performing one of the turns during the holding procedure. The red dashed line in Figure 3 shows the trend of the absolute position error d. Table 1 shows the values of the calculated measures of the aircraft position accuracy and the formula of the trend line. Based on Table 1, the standard deviation σ_d is equal to 218.41 m and the root mean square error $RMSE_P$ is equal to 367.51 m.

Tab. 1

The values of accuracy measure of absolute error of the aircraft position

Accuracy measure	**Value**
Mean absolute position error $\bar{d}$	295.57 m
Standard deviation σ_d	218.41 m
Error $RMSE_P$	367.51 m
Trend line formula	y=8.54x+28.11, where: y- is the absolute error value d x- is the distance of the aircraft from the radar
Maximum value	1733.71 m

Further, in the next stage of the accuracy analysis, the errors in determining the horizontal range ΔR were calculated according to formula 6. Figure 4 shows the error in determining the range coordinate ΔR by the GCA-2000 radar as a function of distance from the radar. Error ΔR includes positive and negative error values, which tell whether the position point determined by the radar is closer or farther from the radar than the point estimated by the GPS receiver. If the value ΔR is positive, then the point determined by the GCA-2000 is farther away from the radar than the GPS point. Similarly, if the value ΔR is negative, the point determined by the radar is closer to the radar than the GPS point. In general, range error values increase as the distance from the radar grows. Similarly to the position error, the range error takes extreme values at a distance of 40-48 km from the radar. The highest error value ΔR is 569.93 m and it occurs when the aircraft exits a sharp 180° turn to enter the approach path. The red line in Figure 4 shows the trend of changes in the error ΔR.

Table 2 shows the values of the accuracy measures of the radar determination of the R_{GCA} coordinate and the formula of the trend line. The average error value $\overline{\Delta R}$ for the whole flight is 138.12 m. The standard deviation of the GCA-2000 radar range error is 95.55 m. This value is within the acceptable range defined by ICAO, which is presented in the papers [15, 16]. Even though a large part of the flight took place at the radar range limit and with rapid changes in azimuth angle, the GCA-2000 radar determined the range with the required accuracy.

In the last stage of the research, the accuracy of azimuth determination $\Delta\beta$ was calculated. Figure 5 shows the polar coordinate error values $\Delta\beta$ as a function of distance from the radar. The presented data refer to the whole flight path considering the positive and negative error values. Negative values mean that the azimuth angle β_{GCA} is greater by an angular value of $\Delta\beta$

from the azimuth angle β_{GPS}. Positive values mean that the azimuth angle determined by the radar is smaller than the azimuth angle β_{GPS}. At the distance of 2-7 km from the GCA-2000 radar, very large deviations of the error values $\Delta\beta$ are visible. These are due to the turn the aircraft made at a distance of 4 km from the radar. An additional factor was the small slant distance of the object from the radar. In this phase of the flight, the aircraft was flying at a high angular velocity relative to the radar antenna. These conditions resulted in the rapid azimuth changes of the aircraft in a short time and consequently lead to very large deviations of the azimuth value determined by the radar in comparison to the actual azimuth at which the aircraft was located. A maximum azimuth error of -2.763° occurred over this distance range. The red line on the graph shows the average error in determining the azimuth $\Delta\bar{\beta}$ by the GCA-2000 radar.

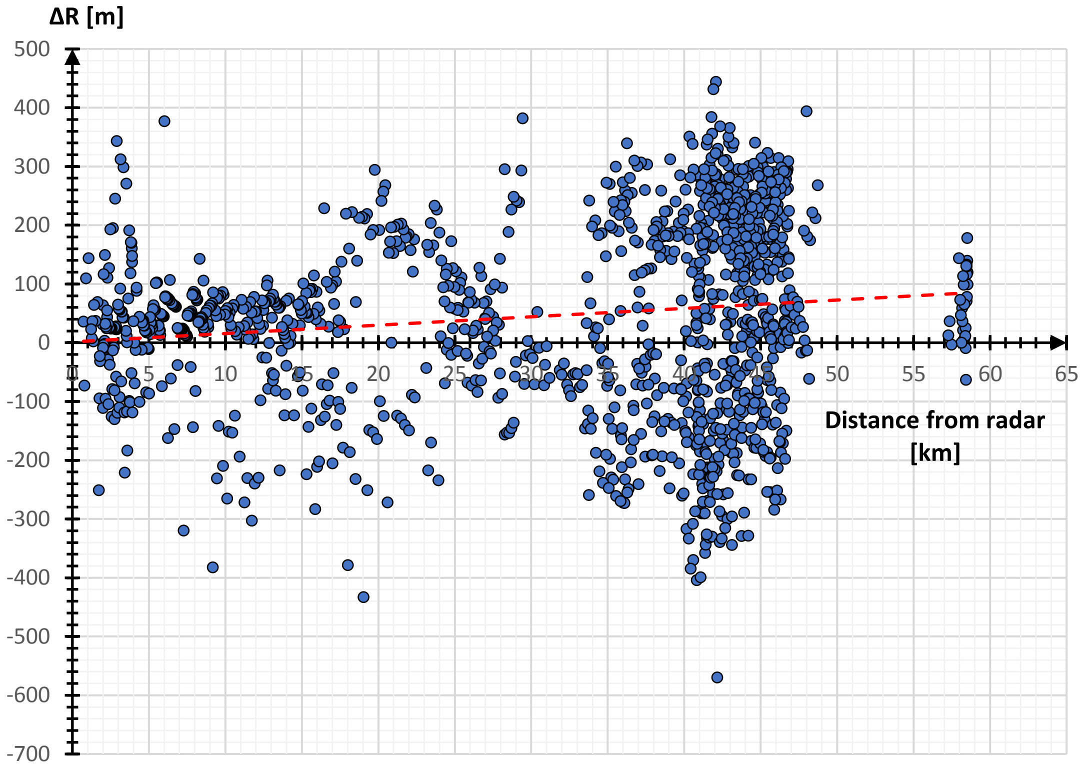

Fig. 4. The absolute error range as a function of distance from the radar

Tab. 2

Values of the accuracy measure of range

Accuracy measure	Value
Mean range error $\overline{\Delta R}$	138.12 m
Standard deviation σ_R	95.55 m
Error $RMSE_R$	138.12 m
Trend line pattern	y= 2.40x+62.81,

	where: y- denotes the error value ΔR x- is the distance of the aircraft from the radar
Maximum value	569.93 m

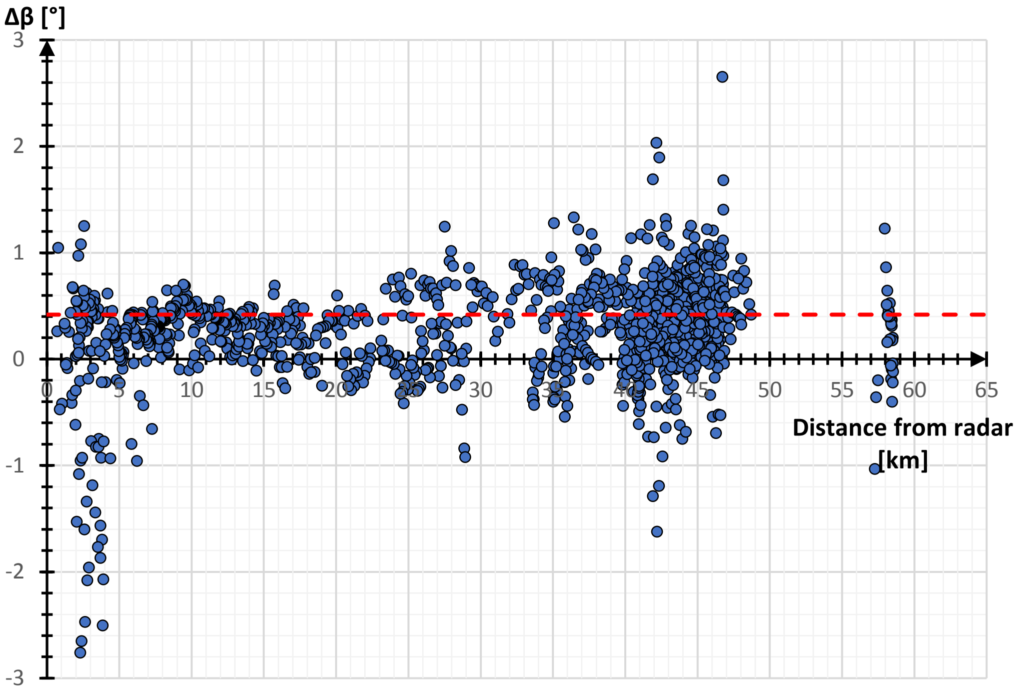

Fig. 5. Absolute error of azimuth as a function of distance from the radar

The values of calculated measures of accuracy of azimuth determination are given in Table 3. The value of the standard deviation of the azimuth error, which is 0.328°, exceeds the acceptable value defined by ICAO by 0.028° [15, 16]. This may be due to the determining influence of deviations in the middle phase of the flight. This was the longest flight phase, wherein most of the points registered by the radar are located. High values of deviations in this phase of flight resulted mainly from a large number of turns performed by the aircraft during holding. They caused rapid changes in the azimuth angle in a very short time. Due to the large number of rapidly changing data, the radar determined the azimuth of the aircraft's position points with less accuracy.

Tab. 3

Values of the accuracy measure of azimuth.

Accuracy measure	**Value**
Mean azimuth error $\overline{\Delta\beta}$	0.408°
Standard deviation σ_{β}	0.328°
Error $RMSE_{\beta}$	0.531°
Trend line pattern	y=0.0093x-0.0296,

	where: y- denotes the error value $\Delta\beta$ x- is the distance of the aircraft from the radar
Maximum value	-2.763°

The average azimuth error $\overline{\Delta\beta}$ for the whole flight equals 0.408°. In turn, the mean square error $RMSE_{\beta}$ is equal to 0.531°. The mean value of the azimuth error $\overline{\Delta\beta}$ determines approximately the axis of symmetry of all parameter results $\Delta\beta$. This may result from an incorrect orientation of the radar to the north.

The values of range error parameters obtained in this work ΔR are worse in comparison with the results shown in the work [5]. In turn, the azimuth errors $\Delta\beta$ have lower values than in the paper [5]. However, the linear trend of changes in the parameter $\Delta\beta$ is the same as in the paper [5]. In the case of comparison of the obtained results to work [7], it can be noticed that both range errors ΔR and azimuth errors $\Delta\beta$ have higher accuracy than in [7].

6. CONCLUSIONS

This paper analyses the accuracy of determining the position parameters of an aircraft using real radar data and GPS satellite data. Measurement data from a GCA-2000 radar and a Thales MobileMapper Pro receiver placed onboard a Diamond DA-40NG aircraft were used in the study. The flight experiment was carried out on 9 December 2021 at the EPDE military airfield in Dęblin. The flight duration was 2 hours, 22 minutes and 2 seconds and ended at the EPDE airport at 11:50:53 UTC. During the flight, the aircraft covered a distance of approximately 440 km at an average speed of approximately 190 km/h. Navigational calculations were performed using the software: MobileMapper Office, TRANSPOL v.2.06 and finally the scripts in Scilab v.6.0.0.

In the course of the experiment, the accuracy of determining the position of the aircraft, as well as the accuracy of determining the azimuth and range by the GCA-2000 radar to the GPS satellite data was tested. The results were expressed in polar and Cartesian coordinates. Based on the performed experimental studies, it was found that:

- the mean positioning error $\bar{d}$ is 295.57 m, and the standard deviation σ_d is equal to 218.41 m, and the error $RMSE_P$ is 367.51 m;
- the average range error $\overline{\Delta R}$ is 138.12 m, and the standard deviation σ_R is equal to 95.55 m, and the error $RMSE_R$ is 138.12 m;
- the average error of determination of the azimuth $\overline{\Delta\beta}$ is 0.408°, and the standard deviation σ_{β} is equal to 0.328°, and the error $RMSE_{\beta}$ is 0.531°.

In addition, the maximum value of the obtained parameters equals to: 1733.71 m for accuracy of position, 569.93 m for accuracy of range, and -2.763° for accuracy of azimuth. This article also shows the trend of changing the accuracy of determining the position, as well as the accuracy of the azimuth and range measurements.

In the future, we plan to use radar data from the AVIA-W radar, located at the Dęblin airport, in the navigation calculations.

References

1. Kulas Małgorzata. 2022. „Processing of the Results of Satellite Inland Positioning Transport Using Mobile Devices and Their Visualization”. *Communications - Scientific Letters of the University of Zilina* 24(1): 36-48. DOI: 10.26552/com.C.2022.1.E36-E48.
2. Novak Andrej, Alena Novak Sedlackova, Anna Stelmach, Doris Novak. 2020. „Safety Implications of GNSS Signal Interference at Zilina Airport”. *Communications - Scientific Letters of the University of Zilina* 22(3): 40-48. DOI: 10.26552/com.C.2020.3.40-48.
3. Czekała Z. 1999. *Parada Radarów*. BELLONA Publishing House. Warsaw. ISBN: 83-11-08806-3 [In Polish: *Parade of radars*].
4. Brzozowski M., M. Pakowski, M. Myszka, M. Michalczewski, U. Winiarska. 2017. "The research of modern radar equipment conducted in the Air Force Institute of Technology by the application of military aircrafts". *Aviation Advances & Maintenance* 1(40): 27-65. DOI: 10.1515/afit-2017-0002.
5. Brzozowski M., M. Myszka, Z. Lewandowski, A. Modrzewski. 2007. "The use of methods of precise determination of the position of aerial objects by means of GPS for the research of the long-range radar RST-12M". *Problemy Techniki Uzbrojenia*, 36(101): 53-62. [In Polish: "Aerial objects precise GPS positioning methods for testing the far range radar ST-12M". *Issues of Armament Technology*].
6. Brzozowski M., Z. Lewandowski. 2009. “Metoda określania rozróżnialności obiektów powietrznych przez urządzenia radiolokacyjne z wykorzystaniem lotów samolotów z zamontowanymi na pokładzie odbiornikami i rejestratorami pozycji”. *Problemy Techniki Uzbrojenia* 38(112): 105-115. [In Polish: "Method of Definition aircrafts distinction by radiolocation devices, using flights of aircrafts witch GPS receivers and position recorders installed on board". *Issues of Armament Technology*].
7. Grzegorzewski M. 2005. "Navigating an aircraft by means of a position potential in three dimensional space". *Annual of Navigation* 9: 1-111.
8. Specht C. 2007. *GPS System*. Bernardinium Publishing House, Pelpin. ISBN: 978-83-7380-469-2. [In Polish: *GPS System*].
9. PANSA website. Available at: https://www.ais.pansa.pl/publikacje/aip-mil/.
10. Thales MobileMapper Pro Technical Specifications. Available at: https://www.thalesnavigation.de/msflash/downloads/MobileMapperCE_EN_A4.pdf.
11. Aviation photos. Available at: https://cdn.jetphotos.com/400/5/99409_1600370828.jpg?v=0.
12. MobileMapper Office. Available at: https://mobilemapper-office.software.informer.com.
13. ASG-EUPOS website. Available at: https://www.asgeupos.pl/index.php?wpg_type=dwnld&sub=gnsstools.
14. Scilab. Available at: https://www.scilab.org/.
15. ICAO website. DOC 8071. 'Manual on Testing of Radio Navigation Aids - Vol. 3 Testing of Surveillance Radar Systems'. First Edition. 1998.
16. EUROCONTROL. "Eurocontrol Standard Document for Radar Surveillance in En-route Airspace and Major Terminal Areas". First Edition. 1997.

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