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## **ACCURACY ANALYSIS OF AIRCRAFT POSITIONING USING GPS DUAL RECEIVERS IN AERIAL NAVIGATION**

**Summary.** This study presents a modified algorithm to determine the accuracy of GPS positioning in aerial navigation. To achieve this, a mixed model with measurement weights was used to determine the resultant value of accuracy of aerial vehicle positioning. The measurement weights were calculated as a function of the number of GPS tracking satellites. The calculations were performed on actual GPS measurement data recorded by two onboard GNSS receivers installed onboard a Cessna 172 aircraft. The flight test was conducted around the military airport in Dęblin. The conducted analyses demonstrated that the developed algorithm improved the accuracy of GPS positioning from 62 to 91% for horizontal coordinates and between 16-83% for the vertical component of the aerial vehicle position in the BLh ellipsoidal frame. The obtained test results show that the developed method improves the accuracy of aircraft position and could be applied in aerial navigation.

**Keywords:** GPS, accuracy, receiver, position errors

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## 1. INTRODUCTION

The GNSS satellite technology already enables precise satellite positioning with the use of 4 global GNSS navigation systems in aerial navigation. These systems include the American GPS, the Russian GLONASS, the European Galileo, and the Chinese BeiDou system [1]. However, following ICAO recommendations [2], currently, only the GPS and GLONASS systems are certified for aviation. The technical specifications, operation, and implementation of the GPS and GLONASS systems have been described in detail in Appendix No. 10 of the Chicago Convention. One of the important elements of ICAO recommendations is the technical standards concerning the quality requirements for GNSS positioning in aerial navigation. This refers in particular to the determination of the accuracy, continuity, availability, and integrity of GNSS parameters in aviation [3]. The most important quality parameter of the GNSS satellite positioning for aerial navigation is accuracy. This parameter defines the difference between the determined coordinates of the aerial vehicle and the reference position of the flight [4]. Thus, accuracy is the essential parameter in using the GNSS satellite technique in aerial navigation.

## 2. SCIENTIFIC KNOWLEDGE ANALYSIS

Research on the GNSS satellite technology in aviation has been conducted in Poland and internationally since the 1990s. However, most of the analyses concerned the application of autonomous positioning methods, especially the SPP code positioning method [5]. As far as international aviation experiments are concerned, they usually employed the SPP method with the use of the GPS or GPS/GLONASS systems [6, 7]. In these cases, the position errors were determined and calculated for the reference trajectory of flight that was determined with the RTK-OTF differential technique. One very interesting solution used in aviation experiments globally was the application of the products of IGS geodesic services in the SPP method, which was described in studies [8-12]. Thus, aviation experiments applied mainly the SP3 precise ephemeris, precise CLK clocks, and the IONEX, DCB, or ANTEX formats. The main objective of this research was to improve the accuracy of autonomous GPS positioning in aerial navigation through the reduction or the application of new models of systematic errors in the SPP code method. On the other hand, aviation research conducted in Poland focused mainly on the wide testing of various GNSS receivers in aerial navigation [13], to determine the actual accuracy of positioning of aerial vehicles. The analyses were conducted both in real-time and in the post-processing mode [14]. In these studies, the resulting coordinates of an aerial vehicle obtained with the SPP method were compared with the SBAS/EGNOS method, the differential code method DGPS or the phase differential technique RTK-OTF [13, 15].

The analysis of the state of knowledge leads to the following conclusions:

- Numerous aviation tests were conducted to determine the accuracy of aircraft positioning using the SPP code method;
- Tests were conducted both in real-time and in the post-processing mode, using various classes of GNSS satellite receivers;
- The conducted research demonstrates the existence of a problem, which is the low positioning accuracy when the SPP code method is used;
- Further flight tests should be conducted using the GNSS satellite technique, particularly the SPP code method.
- This allows us to conclude that:

- It is necessary to develop new mathematical algorithms that will improve the determination of the accuracy parameter;
- Further research on improving the navigation solutions that employ positioning from the SPP method is necessary;
- It is possible to apply a positioning navigation solution that is based on at least 2 GNSS satellite receivers.

This study aims to develop a modified algorithm that will enable the improvement of the determination of the accuracy parameter using the SPP code method. To this end, the resultant positioning accuracy was calculated for 2 GNSS satellite receivers. The calculations were based on a mixed model that combined the obtained single accuracy values determined for an individual GNSS receiver. The developed algorithm was tested for GPS data using the SPP code method. It proved to be universal; hence, it may be used in the future for other GNSS satellite systems in aerial navigation.

The article consists of 7 sections: 1 – Introduction, 2 – Analysis of the state of knowledge, 3 – Research method, 4 – Research test, 5 – Test results, 6 – Discussion, and 7 – Conclusions. The bibliography is presented at the end.

### 3. RESEARCH METHOD

The basic algorithm for the determination of the resultant accuracy value is based on the mixed model in the following form:

- for the B geodesic latitude component:

$$dB = \alpha \cdot dB_{SPP,Rx1} + \beta \cdot dB_{SPP,Rx2} \quad (1)$$

- for the L geodesic longitude component:

$$dL = \alpha \cdot dL_{SPP,Rx1} + \beta \cdot dL_{SPP,Rx2} \quad (2)$$

- for the h ellipsoidal height component:

$$dh = \alpha \cdot dh_{SPP,Rx1} + \beta \cdot dh_{SPP,Rx2} \quad (3)$$

where:

$\alpha$  – measurement weight for receiver 1,

Rx1 – designation of receiver 1,

$\beta$  – measurement weight for receiver 2,

Rx2 – designation of receiver 2,

$dB_{SPP,Rx1}$  – positioning accuracy along the B axis from receiver 1 for the SPP code method,

$dB_{SPP,Rx2}$  – positioning accuracy along the B axis from receiver 2 for the SPP code method,

$dL_{SPP,Rx1}$  – positioning accuracy along the L axis from receiver 1 for the SPP code method,

$dL_{SPP,Rx2}$  – positioning accuracy along the L axis from receiver 2 for the SPP code method,

$dh_{SPP,Rx1}$  – positioning accuracy along the h axis from receiver 1 for the SPP code method,

$dh_{SPP,Rx2}$  – positioning accuracy along the h axis from receiver 2 for the SPP code method,

(dB, dL, dh) – resultant accuracy values (position errors for the BLh components).

The values of measurement weights ( $\alpha$ ,  $\beta$ ) were expressed in the form:

$$\alpha=1/NS_{SPP,Rx1} \text{ and } \beta=1/NS_{SPP,Rx2} \quad (4)$$

where:

$NS_{SPP,Rx1}$  – defines the number of tracked GPS satellites used in the positioning of the aerial vehicle for the SPP code method for receiver 1,

$NS_{SPP,Rx2}$  – defines the number of tracked GPS satellites used in the positioning of the aerial vehicle for the SPP code method for receiver 2.

The applied algorithm (1-4) was tested and verified for kinematic GPS in a flight experiment. The experiment is described in Section 4.

#### 4. RESEARCH TEST

The research test was divided into two stages. The first stage consisted of a test flight with a Cessna 172N aircraft around the military airport in Dęblin. The test flight lasted from 13:47:20 hours to 16:27:00 hours according to GPS system time. The horizontal and vertical trajectories of the Cessna 172N aircraft are presented in Figures 1 and 2. The B coordinate changed from 51.476977 to 53.299673°, while the L coordinate changed between 21.85564 and 23.305957°. The change in the h coordinate ranged from 149.82 to 1271.30 m. Two navigation Thales Mobile Mapper receivers were installed onboard the aircraft to determine its position using the SPP code method. Additionally, it was possible to determine the aircraft positioning accuracy for a single receiver, that is, to determine the ( $dB_{SPP,Rx1}$ ,  $dL_{SPP,Rx1}$ ,  $dh_{SPP,Rx1}$ ) parameters for receiver 1, and the ( $dB_{SPP,Rx2}$ ,  $dL_{SPP,Rx2}$ ,  $dh_{SPP,Rx2}$ ) parameters for receiver 2. Following Formula (1), receiver 1 was marked as Rx1 and receiver 2 as Rx2. Thus, it may be stated that the single positioning accuracy for the BLh ellipsoidal coordinates was determined separately for receivers 1 and 2.

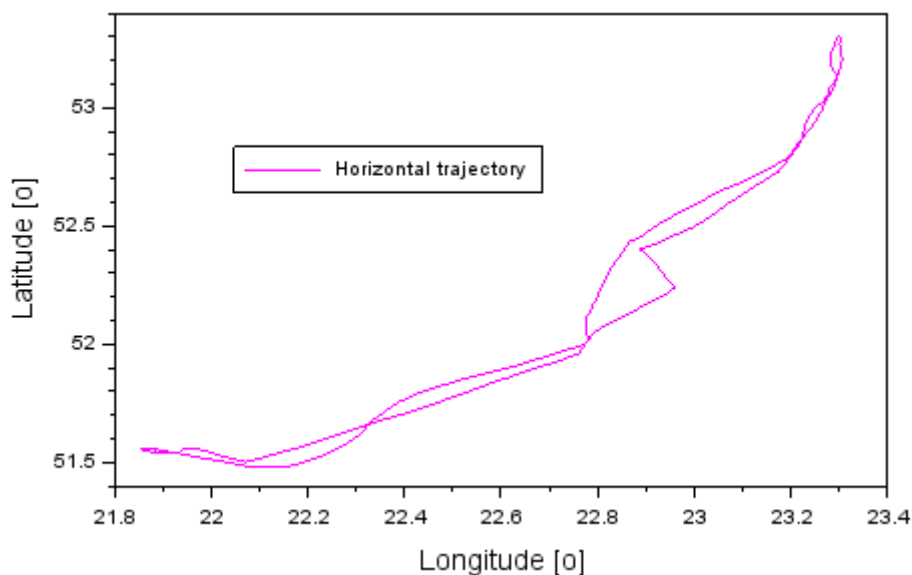


Fig. 1. Horizontal trajectory of the aircraft

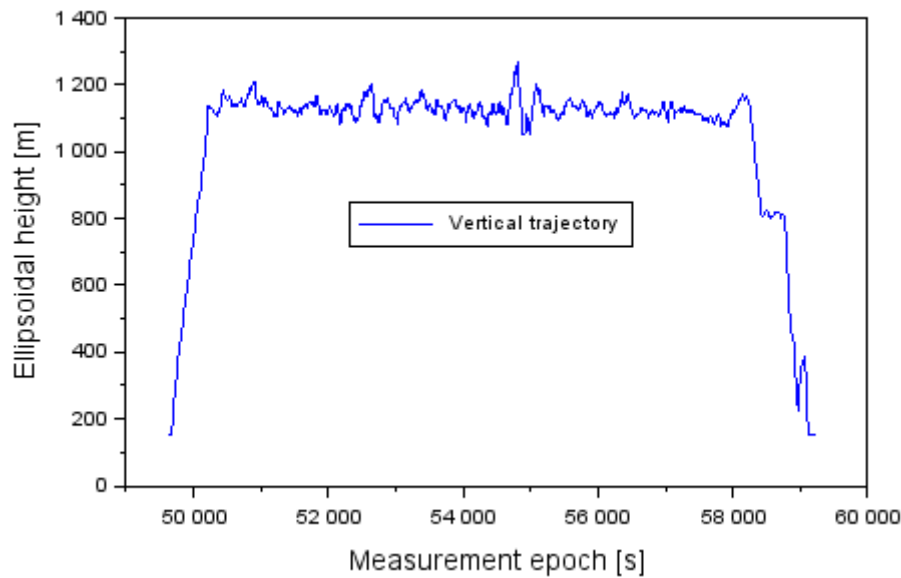


Fig. 2. Vertical trajectory of the aircraft

The second stage of the experiment consisted of collecting the navigation data recorded by both satellite receivers, followed by processing, transmitting and cataloguing these data on a portable computer for further data processing. All GPS navigation data were saved in one folder on the portable computer. For the navigation data are included: the coordinates of the aircraft determined using the SPP code method, the reference coordinates of the flight calculated using the RTK-OTF differential technique, and the single accuracy results obtained for each of the receivers Rx1 and Rx2. Apart from that, navigation data on the number of GPS satellites tracked by receivers Rx1 and Rx2 were recorded.

The third stage of the research consisted of the development and implementation of the mathematical algorithm (1-4) in the given programming language. In this analysed case, numerical calculations were performed in the Scilab v.6.0.0 language environment [16]. Calculations were performed for a total of 9581 measurement epochs, with 1s steps. Measurement weights were calculated from formula (4) for both receivers Rx1 and Rx2. The results of the numerical calculations together with their graphic representations and tables (Not presented) are presented in Section 5.

## 5. RESEARCH RESULTS

The presentation of research results begins with presenting the numbers of tracked GPS satellites for the Rx1 and Rx2 receivers for the solution from the SPP code method. Figure 3 shows the results of the  $(NS_{SPP,Rx1}, NS_{SPP,Rx2})$  parameters. Both Thales Mobile Mapper receivers recorded GPS signals from 5 to 8 satellites during the flight test. For most of the flight, both satellite receivers tracked at least 7 GPS satellites.

Next, Figure 4 presents the measurement weights  $(\alpha, \beta)$  calculated from equation (4). The values of measurement weights for both Thales Mobile Mapper receivers range from 0.125 to 0.200. However, the value of the  $\alpha$  weight coefficient is 0.141 and of the  $\beta$  coefficient 0.137. The values of the weight coefficients  $(\alpha, \beta)$  increase with the decreasing number of GPS satellites tracked by the Rx1 and Rx2 satellite receivers. This is a reverse relationship, so when the

number of GPS satellites tracked by the Rx1 and Rx2 receivers increases, the weight coefficients ( $\alpha$ ,  $\beta$ ) decrease.

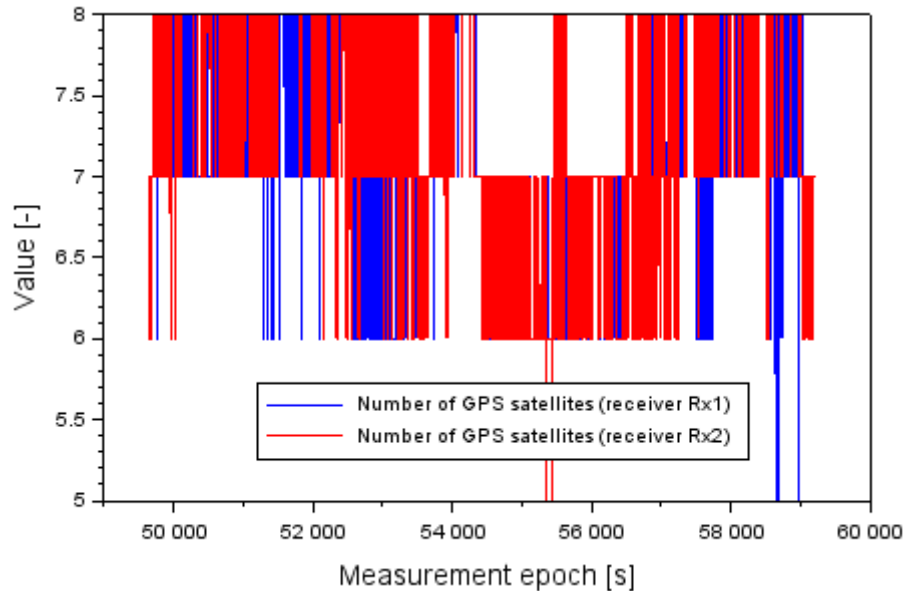


Fig. 3. Number of GPS satellites tracked during flight test

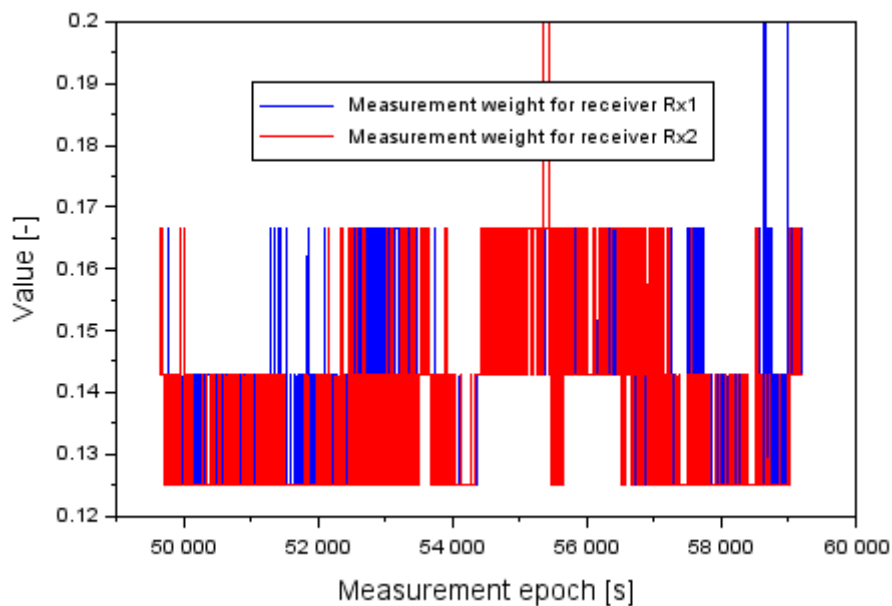


Fig. 4. Values of measurement weight ( $\alpha$ ,  $\beta$ )

Figures 5 and 6 show the results of single accuracy values for both Rx1 and Rx2 satellite receivers. Hence, Figure 5 presents the results of position errors obtained for the Rx1 receiver. The values of the  $dB_{SPP,Rx1}$  parameter range from  $-7.71$  to  $+3.27$  m, while values of the  $dL_{SPP,Rx1}$  parameter range from  $-5.30$  to  $+5.07$  m. Finally, the results of the  $dh_{SPP,Rx1}$  parameter range from  $-12.62$  to  $+5.25$  m. Figure 6 presents the results of position errors obtained for the Rx2 receiver. The values of the  $dB_{SPP,Rx2}$  parameter range from  $-9.61$  to  $+0.67$  m, while values of the  $dL_{SPP,Rx2}$  parameter range from  $-6.91$  to  $+5.73$  m. Finally, the results of the  $dh_{SPP,Rx2}$  parameter range from  $-16.67$  to  $+11.17$  m. As one may notice in Figures 5 and 6, the lowest

positioning accuracy from the SPP solution is noted for the ellipsoidal height component. On the other hand, the highest positioning accuracy was noticed along the L axis.

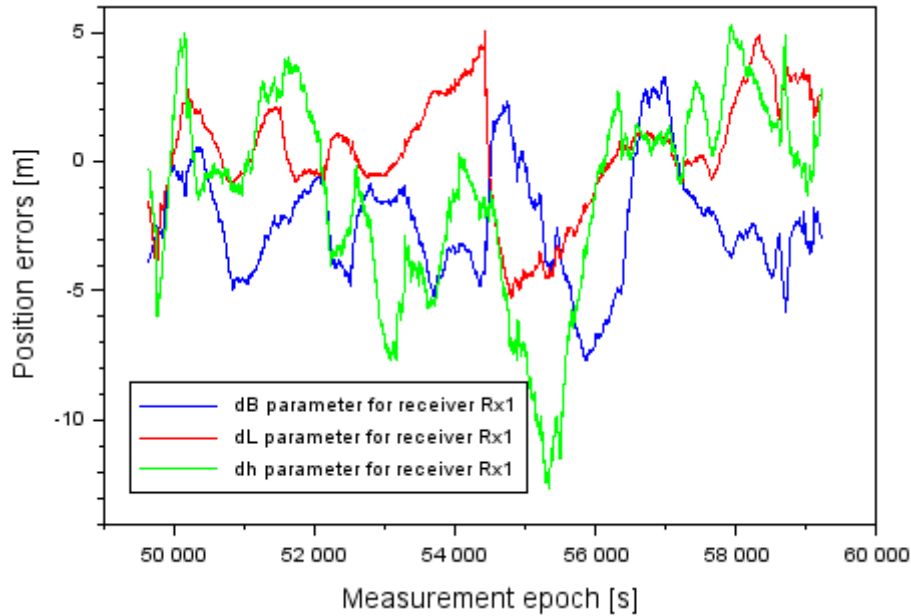


Fig. 5. Values of position errors for receiver  $R_{x1}$

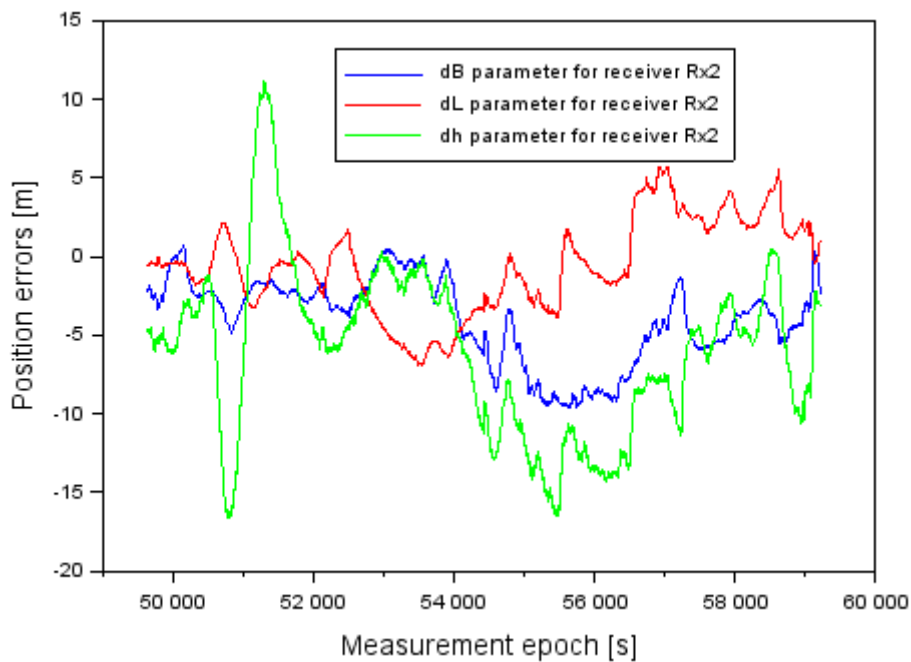


Fig. 6. Values of position errors for receiver  $R_{x2}$

The results of single positioning accuracy presented in Figures 5 and 6 are followed by Figure 7, which illustrates the final results of the resultant accuracy value according to the algorithm (1-3). The positioning accuracy for the B component ranged from -2.65 to +0.01 m. For the L component, the accuracy ranged from -1.38 to +1.23 m, while for the vertical component h, it falls into the range of -5.13 to +2.01 m.

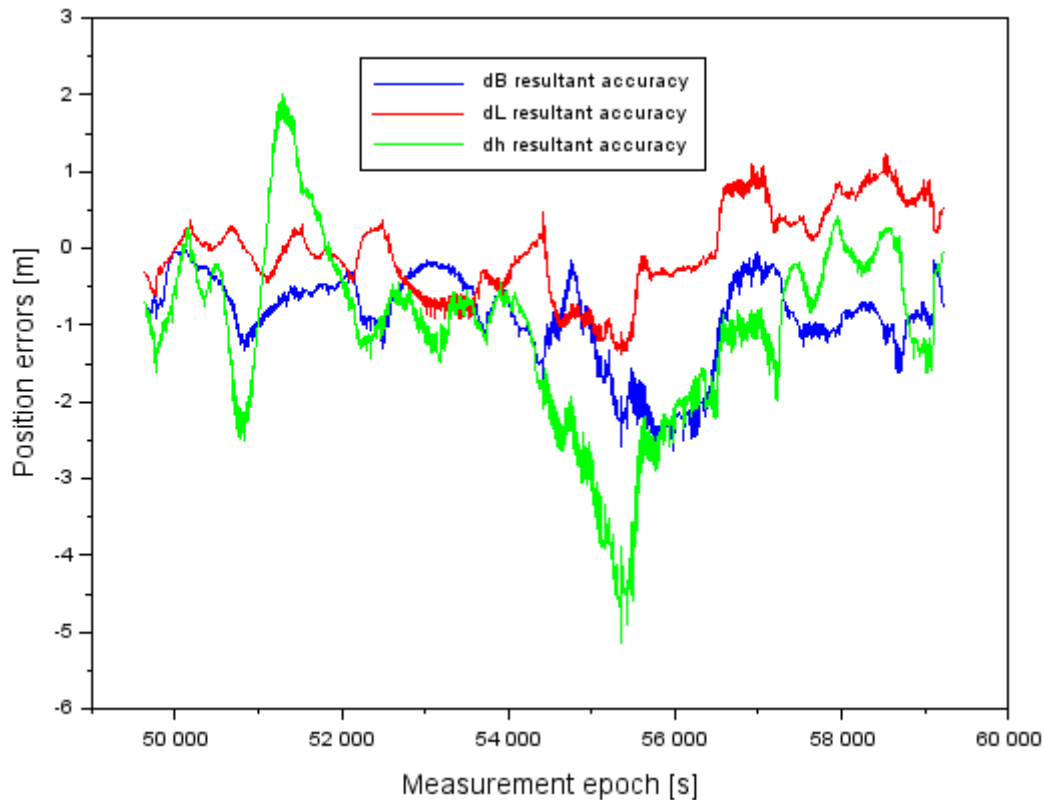


Fig. 7. Values of resultant accuracy of aircraft position

## 6. DISCUSSION

The discussion focuses on two research threads. First, the importance of the proposed mathematical solution (1-4) for single accuracy values from single satellite receivers was highlighted. The second research thread elaborates on the significance of the contribution of this study to the current state of knowledge.

In the first part of the discussion, the authors compared the obtained values of resultant accuracy with single accuracy values for both GNSS receivers. To do this, the mean values of aircraft positioning accuracy were compared. Figure 8 presents a comparison of the (dB,  $dB_{SPP,Rx1}$ , and  $dB_{SPP,Rx2}$ ) parameters in the form of absolute values. The mean value of the dB parameter equals 0.9 m, while for the  $dB_{SPP,Rx1}$  parameter, it is 2.4 m, and finally, for the  $dB_{SPP,Rx2}$  parameter, it equals 4.0 m. Based on these values, one may conclude that the values of the dB parameter are decidedly lower than the results of  $dB_{SPP,Rx1}$  and  $dB_{SPP,Rx2}$ . Therefore, the resultant accuracy dB is higher than the results for parameters  $dB_{SPP,Rx1}$  and  $dB_{SPP,Rx2}$ . It may be stated that the resultant accuracy dB improved by over 62% in comparison to the results of the  $dB_{SPP,Rx1}$  parameter, and by 78% compared to the results of the  $dB_{SPP,Rx2}$  parameter.

Figure 9 presents a comparison of the (dL,  $dL_{SPP,Rx1}$  and  $dL_{SPP,Rx2}$ ) parameters in the form of absolute values. The mean value of the dL parameter equals 0.1 m, of the  $dL_{SPP,Rx1}$  0.4 m, while for the  $dL_{SPP,Rx2}$ , it is 0.6 m. These values lead to the conclusion that the values of the dB parameter are decidedly lower than the results of  $dL_{SPP,Rx1}$  and  $dL_{SPP,Rx2}$ . Thus, the resultant accuracy dL is higher than the results for parameters  $dL_{SPP,Rx1}$  and  $dL_{SPP,Rx2}$ . One may conclude that the resultant accuracy dL improved by over 85% in comparison to the results for the  $dL_{SPP,Rx1}$  parameter, and by 91% compared to the results of the  $dL_{SPP,Rx2}$  parameter.



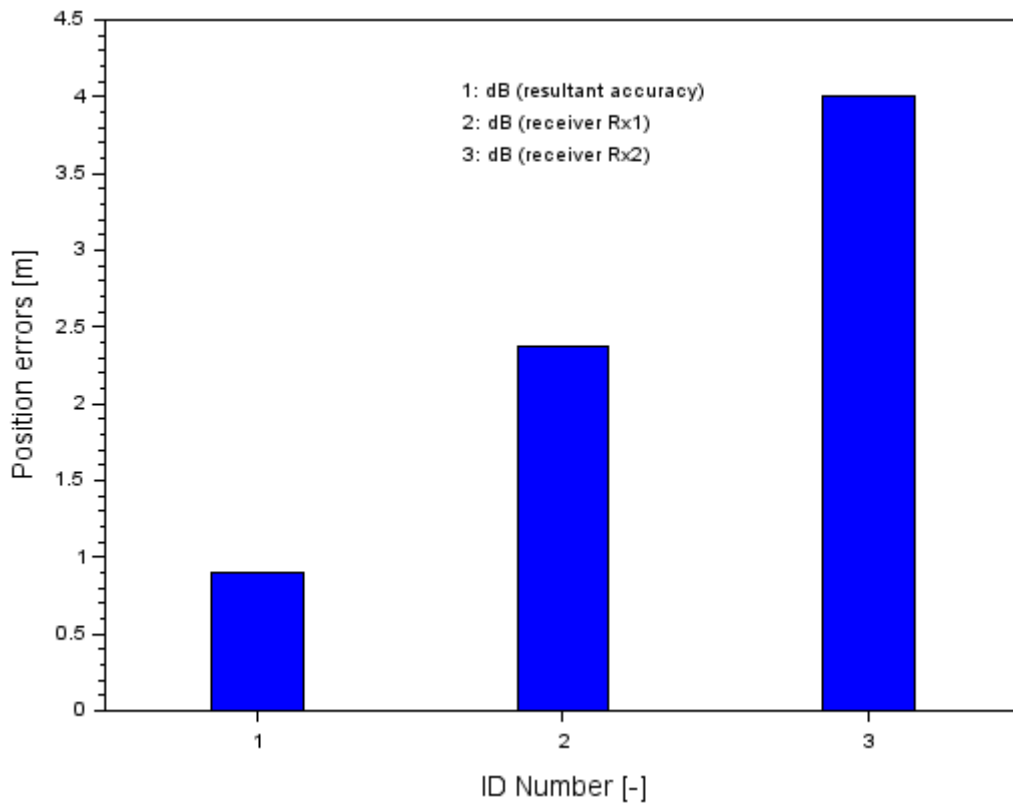


Fig. 8. Comparison of position errors along the B axis

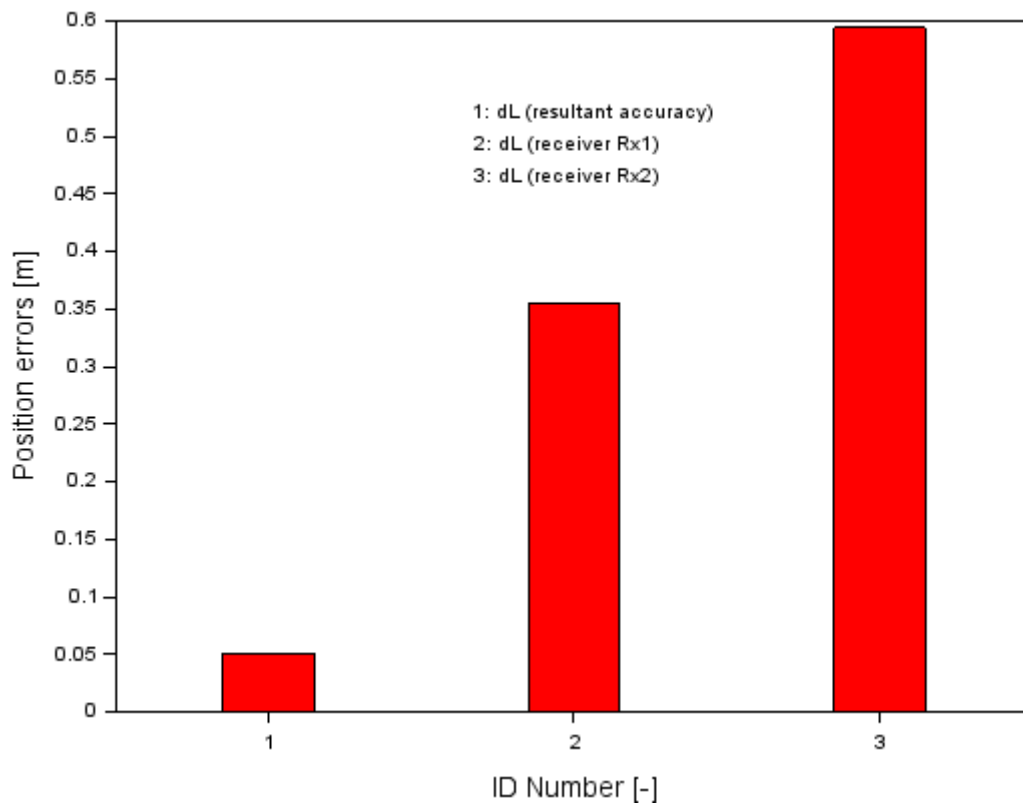


Fig. 9. Comparison of position errors along the L axis

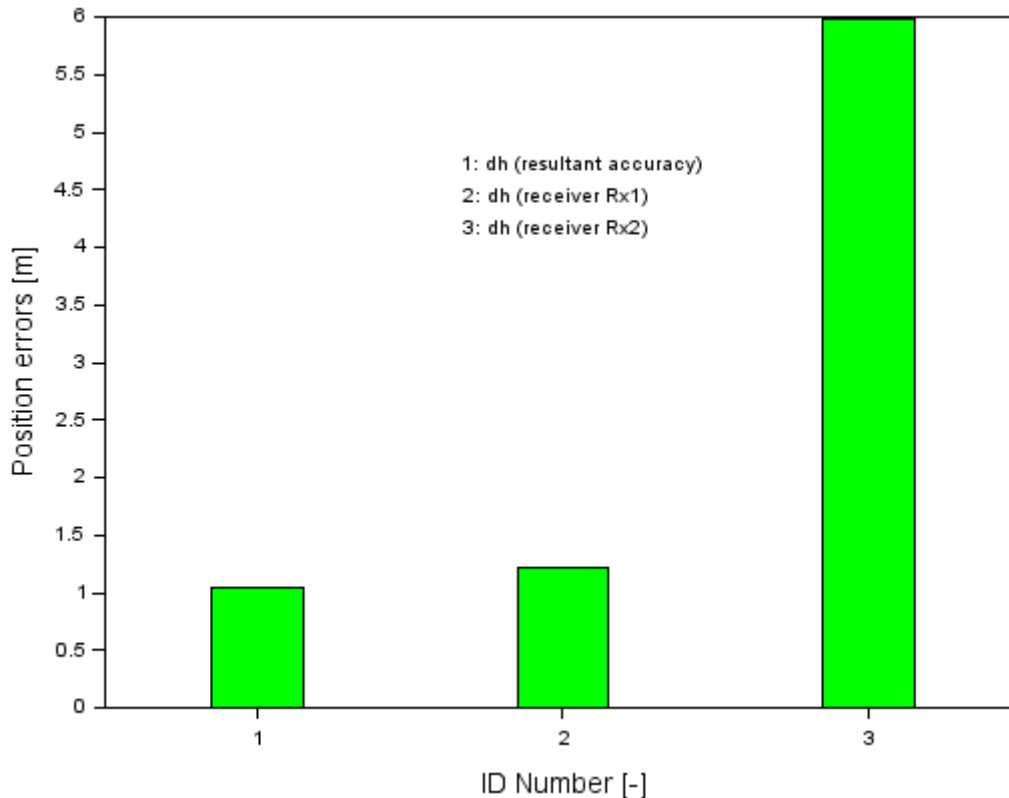


Fig. 10. Comparison of position errors along the h axis

Finally, Figure 10 presents a comparison of the ( $dh$ ,  $dh_{SPP,Rx1}$ , and  $dh_{SPP,Rx2}$ ) parameters in the form of absolute values. The mean value of the  $dh$  parameter equals 1.0 m, for the  $dh_{SPP,Rx1}$ , it is 1.2 m, while for the  $dh_{SPP,Rx2}$  parameter, it equals 6.0 m. These values lead to the conclusion that the values of the  $dh$  parameter are decidedly lower than the results of  $dh_{SPP,Rx1}$  and  $dh_{SPP,Rx2}$ . Hence, the resultant accuracy  $dh$  is higher than the results for parameters  $dh_{SPP,Rx1}$  and  $dh_{SPP,Rx2}$ . This leads to the conclusion that the resultant accuracy  $dh$  improved by over 16% in comparison to the results of the  $dh_{SPP,Rx1}$  parameter, and by 83% compared to the results of the  $dh_{SPP,Rx2}$  parameter.

The second part of the discussion shows the contribution of this study to the current state of knowledge. The obtained positioning accuracy results are decidedly better than those presented in these publications [13-15]. Similarly, the obtained research results are comparable to or better than the results provided in some works [6-12]. It may be concluded that the proposed navigation solution for mathematical equations (1-4) improves the accuracy of aircraft positioning using the SPP code method in the GPS satellite system. Thus, the algorithm (1-4) presented here may provide an interesting solution for determining the accuracy parameter in aerial navigation.

## 7. CONCLUSIONS

This article presents the results of research on the determination of GPS positioning accuracy in aerial navigation. Thus, a modified algorithm was created to determine the accuracy parameter of a set of two GNSS satellite receivers installed onboard an aircraft. The functioning of the algorithm was tested on actual GPS measurement data using the SPP code positioning

method. The analyses were conducted on GPS satellite data from two Thales Mobile Mapper onboard receivers installed in a Cessna 172 aircraft, which performed a test flight over the airport in Dęblin. The performed calculations revealed that:

- the resultant positioning accuracy for the B component improved by 62-78% compared to the single accuracy results for a single GNSS receiver;
- the resultant positioning accuracy for the L component improved by 85-91% compared to the single accuracy results for a single GNSS receiver;
- the resultant positioning accuracy for the h component improved by 16-83% compared to the single accuracy results for a single GNSS receiver.

The obtained test results demonstrated that the algorithm applied to improve the GPS positioning accuracy is correct and could be used in aviation and navigation operations.

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