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DESIGNING PITCH CURVES OF NON-CIRCULAR GEARS

Summary. The paper examines the design and generation of gear drives with non-circular gears. Gearings with a changing transmission gear ratio are used for the purposes of practical experimentation, even though “standard” gearings with a constant transmission gear ratio are used more often. In this paper, the author presents the design for the shape of a pitch outline for a specific requirement, that is, continuous change in gear ratio during one rotation. The gearing is designed such that the pitch curve is composed of an ellipse. The article also presents some kinematic properties of the designed non-circular gearing.
Keywords: gear; non-circular; ellipse; pitch outline; variable transmission ratio

1. INTRODUCTION

Gearboxes belong to the most used transmission mechanisms. The history of gears is probably as old as civilization itself. The earliest description of gears was written in the fourth century BCE by Aristotle, who wrote that the “direction of rotation is reversed when one gear wheel drives another gear wheel”. In practice, the most commonly used “standard” toothed gears are characterized by a constant gear number and circular wheel shape. Non-circular gears are not well known, even though the idea behind them originates from the pioneers of engineering thought. For example, such gears were sketched by Leonardo da Vinci (Fig. 1) [1], while, in the late 19th century, Franz Reuleaux ordered a series of non-circular gear

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models from the Gustav Voigt Mechanische Werkstatt in Berlin to help with the study kinematics. These historical gears comprised simplified tooth shapes and, for this reason, the meshing conditions were not always correct.

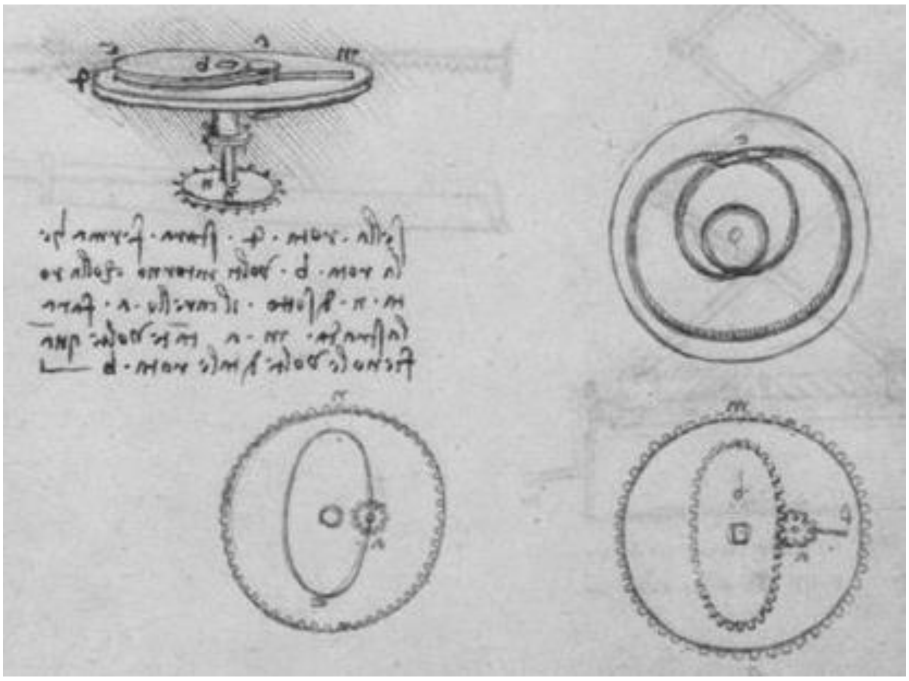

Fig. 1. Sketch by Leonardo da Vinci [1]

Non-circular gears are presented as a curiosity from the gear industry history, due to their complex design and manufacturing difficulties. Nowadays, performance modelling and simulation software, advanced CNC machine tools and non-conventional manufacturing technologies enable non-circular gear design and manufacture.

As mechanisms used to generate variable motion laws, in comparison with cams, linkages, variable transmission belts, Geneva mechanisms and even electrical servomotors, non-circular gears are remarkable due to their advantages, such as the ability to produce variable speed movements in a simple, compact and reliable way, the lack of gross separation or decoupling between elements, fewer parts in the design phase, and the ability to produce high strength-to-weight ratios [2-4].

The applications of non-circular gears include textile industry machines, for improving machine kinematics resulting in process optimization [5,6], window shade panel drives, for introducing vibrations that interfere with natural oscillations and cancelling them out [7], high-torque hydraulic engines for bulkhead drives [8-11], and mechanical presses, for optimizing work cycle kinematics. They are also used as high-power starters, mechanical systems (providing progressive torque for easier start-ups of machines, where progressive torque helps to overcome the start-up inertia) and forging machines (optimizing the work cycle parameters and reducing pressure dwell time). The use of non-circular gears in industry certifies their effective performance, while prompting new ideas for improved working conditions. Non-circular gears are also used on oval gear flowmeters [12], which are categorized as positive displacement flow technology devices. Positive displacement flow technology allows for precise flow measurements of most clean liquids, regardless of media conductivity.

The generation of non-circular gears is usually developed from a hypothetical basis, such as the law of driven gear motion, variation in the gear transmission ratio, and the design of the driving gear pitch curve [13-14]. The studies for the design and manufacture of non-circular helical gears are highly limited.

The first step in the non-circular gear virtual design process is the generation of the conjugate pitch curves, starting from a predesigned law of motion for the driven element or a predesigned geometry for the driving gear pitch curve. The current paper is dedicated to this problem.

2. CONDITIONS OF PROPER MESHING

The industry standard involute tooth shape has been chosen for use in non-circular gears. Thus, the existing involute gearing standards and methods can be adopted and applied. The design of correct meshing must be based on the basic conditions that are imposed on gearing. The tooth profiles, which form a shape bond, must have a designed continuous meshing. Otherwise, in order to transmit a uniform rotary motion from one shaft to another by means of gear teeth, the normals to the profiles of these teeth at all points of contact must pass through a fixed point (point P in Fig. 2.) in the common centre line (O_1O_2 in Fig. 2.) of the two shafts. The fixed point is, of course, the pitch point; and, for involutes, the normals (line n in Fig. 2.) fall onto the line of action. The instantaneous normals to the profiles of these teeth at all points of contact must pass through a fixed point in the common centre line of the two shafts to transmit any rotary motion from one shaft to another by means of gear teeth t. The pitch curves (k_1 and k_2 in Fig. 2.) correspond to the pitch circles in “standard” circular gears.

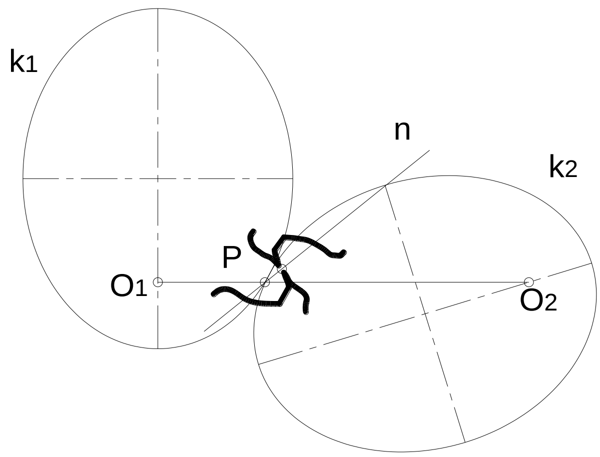

Fig. 2. Condition of the meshing of non-circular gears

Fig. 3 depicts non-circular gears as two rollers rolling together without slippage, provided there it is no addendum modification and the nominal axle distance (parameter a in Fig. 3.) is used. Roll lines (pitch curves) are divided into z parts, which are p long, where z is the gear’s number of teeth and p is the pitch. The gear is represented by two pitch curves k_1 and k_2 with centres in points O_1 and O_2. The variable pitch curve radii, $r_1(\varphi)$ and $r_2(\varphi)$, of the non-circular gears are determined by the required course of the transmission ratio i and therefore the ram kinematics:

$$i(\varphi) = \frac{d\varphi}{d\psi} = \frac{1}{\psi'(\varphi)} = -\frac{r_2(\varphi)}{r_1(\varphi)} = \frac{\omega_1(\varphi)}{\omega_2(\varphi)} \tag{1}$$

where $\omega_1(\varphi)$ and $\omega_2(\varphi)$ are the angular velocity functions for Gears 1 and 2, respectively.

With a given constant centre distance:

$$a = constant = r_1(\varphi) + r_2(\varphi) \tag{2}$$

the transmission function $\psi'(\varphi)$ describes the relation between the pitch curves of the non-circular gears. In the case of an aperiodic non-circular gear, angular positions of the members are limited [17-19]. Usually, a specific ratio function is used, for example, a logarithmic function.

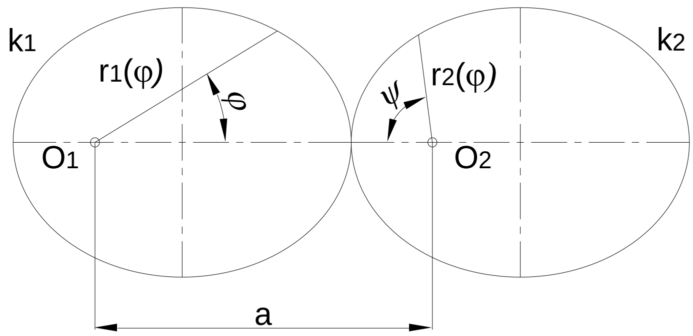

Fig. 3. Example of non-circular pitch curves

3. ECCENTRIC ELLIPTICAL GEARING

3.1. Characteristic of non-circular gearing

The generation of this non-circular gear was developed from a hypothetical basis, such as the law of driven gear motion, variation in the gear transmission ratio and the design of the driving gear pitch curve. This model of non-circular gear was designed for a variable transmission ratio in the range $u=0.25$ to 4.0. This transfer should be formed by two identical wheels with the number of teeth $z_1=z_2=40$ and gearing module $m_n=4$ mm, where distance $a=160$ mm, and for one direction of rotation.

3.2. Design of pitch curves

The first step in the non-circular gear design process is the generation of the pitch curves, starting from a predesigned law of motion for the driven element or a predesigned geometry for the driving gear pitch curve. For a non-standard gearing, an eccentric elliptical gear drive with a continuously changing transmission gear ratio was applied; that is, the ellipse was used as the pitch curve (Fig. 4.). For the given distance, the pitch ellipse had a large half-axis $a_e=80$ mm, which is half of the axial distance. The position of ellipse focus was determined by considering the desired continuously changing transmission gear ratio. For the given variable transmission ratio in the range u=0.25 to 4.0, the position of the ellipse focal point (centre point O of rotation) is determined by the ratio lengths x_1: x_2, which are equal to 1:4. The second half-axis, $b_e=64$ mm, is determined by the distance from the focus point $a_e=80$ mm for the transmission ratio $u=1$.

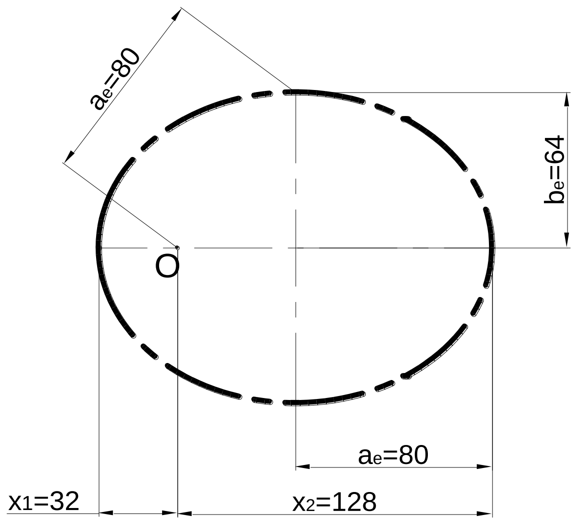

Fig. 4. Pitch ellipse for gear ratio u=0.25 to 4

In this case, one of the conditions of a correct mesh is that the measurements of the pitch on the ellipse pitch must be kept constant. A geometric separation of the pitch ellipse into 40 identical sections (the number of teeth $z_1=z_2=40$) is mathematically much more difficult than in the case involving standard gear pitch circles.

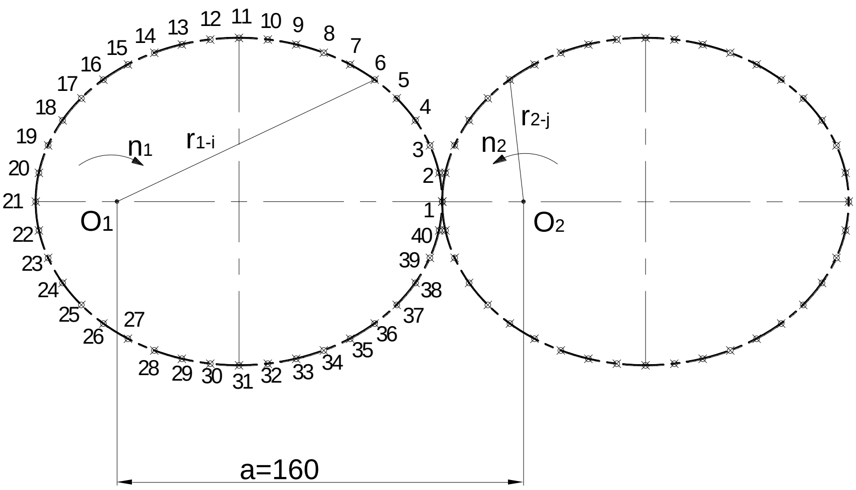

Fig. 5. Design of the non-circular pitch curves

Fig. 5 presents the pitch ellipses of a designed eccentric elliptical gear drive with a continuously changing transmission gear ratio for given parameters. Torque transmission ensures shape-bonding between the meshing gears. The gearing consists of two identical gears. The toothed number is shown for the drive wheel; for the driven wheel, this numbering is the same. Wheels are designed for only one direction of rotation. The pitch ellipses must meet the condition that, for each tooth, the sum of the radii is equal to the axial distance (Tab. 1.).

$$r_{1-i} + r_{2-j} = a = 90\ mm \tag{3}$$

where $r_{1\text{-}i}$ and $r_{2\text{-}j}$ are the radii of mesh points.

3.3. Kinematic properties of non-circular gearing

In pursuit of kinematic ratios for the proposed gearings, we assumed the right mesh conditions. Kinematic conditions were processed for the drive wheel (centre of rotation at point O_1) and the driven wheel (with the centre of rotation at point O_2). On the relevant graph, the two gears are shown in a state of kinematic dependence (initially, on the horizontal axis of the wheel teeth). In Tab. 1, the dimensions of the spacing radii at the individual points of contact are designated as $r_{1\text{-}i}$, respectively, while $r_{2\text{-}j}$, where Index *1* applies to the drive wheel, Index *2* for the driven wheel, and Index *i* or *j* corresponds to the order number of the engaging tooth (Fig. 5) at one turn of the drive and driven wheel.

Tab. 1.

The kinematic properties of elliptical gearing

Meshing teeth input - output	Radius of mesh points		Centre distance $a=r_{1\text{-}i}+r_{2\text{-}j}$ (mm)	Transmission ratio $u_i=r_{2\text{-}j}/r_{1\text{-}i}$	Rotational speed $\omega_{2i}=\omega_1/u_i$ (s^{-1})
	$r_{1\text{-}i}$ (mm)	$r_{2\text{-}j}$ (mm)			
1-21	128	32	160	**0.250**	400
02-20	127.25	32.75	160	0.257	388. 550
03-19	125.08	34.92	160	0.279	358. 190
04-18	121.67	38.33	160	0.315	317. 428
05-17	117.24	42.76	160	0.365	274.181
6-16	112.03	47.97	160	0.428	233.542
07-15	106.24	53.76	160	0.506	197.619
08-14	100.01	59.99	160	0.600	166.711
09-13	93.5	66.5	160	0.711	140.601
10-12	86.79	73.21	160	0.844	118.549
11-11	80	80	160	**1**	100
12-10	73.21	86.79	160	1.185	84.353
13-9	66.5	93.5	160	1.406	71.123
14-8	59.99	100.01	160	1.667	59.984
15-7	53.76	106.24	160	1.976	50.602
16-6	47.97	112.03	160	2.335	42.819
17-5	42.76	117.24	160	2.742	36.472
18-4	38.33	121.67	160	3.174	31.503
19-3	34.92	125.08	160	3.582	27.918
20-2	32.75	127.25	160	3.885	25.737
21-1	32	128	160	**4**	25
22-40	32.75	127.25	160	3.885	25.737
23-39	34.92	125.08	160	3.582	27.918
24-38	38.33	121.67	160	3.174	31.503
25-37	42.76	117.24	160	2.742	36.472
26-36	47.97	112.03	160	2.335	42.819

Fig. 6 represents the course of a continuously changing gear ratio in one mesh generated by an elliptical gear, which continuously varies in the range from $u=0.25$ to $u=1.0$ until $u=4.0$, and back. Thus, the gear ratio changes over the duration of one revolution. A gear ratio value that is less than *1.0* signifies that this is an overdrive, while a gear ratio value greater than *1.0* signifies a speed reduction.

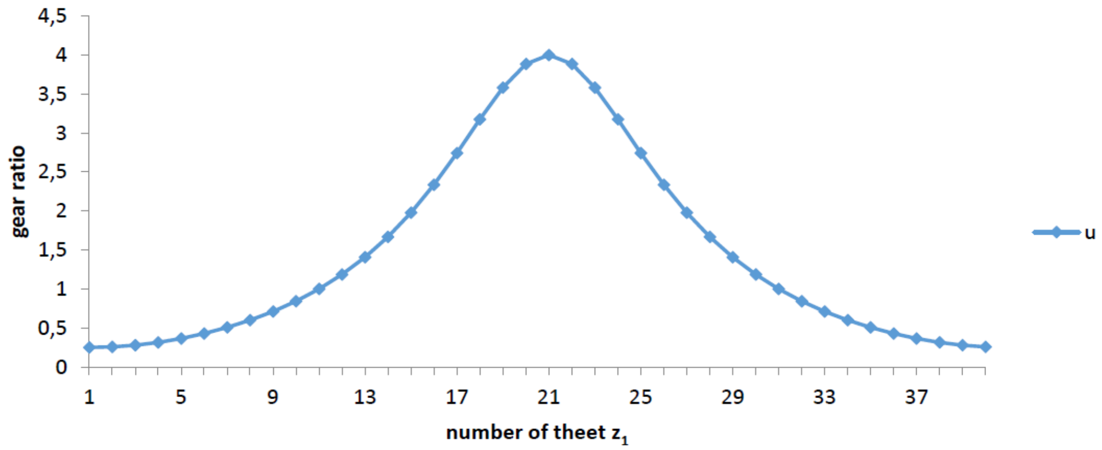

Fig. 6. Transmission gear ratio

Fig. 7 shows the progress of the meshing radii at the individual points of contact, designated as r_{1-i}, respectively, while r_{2-i}, where Index *1* applies to the drive wheel, Index *2* to the driven wheel, and Index *i* or *j* corresponds to the order number of the tooth.

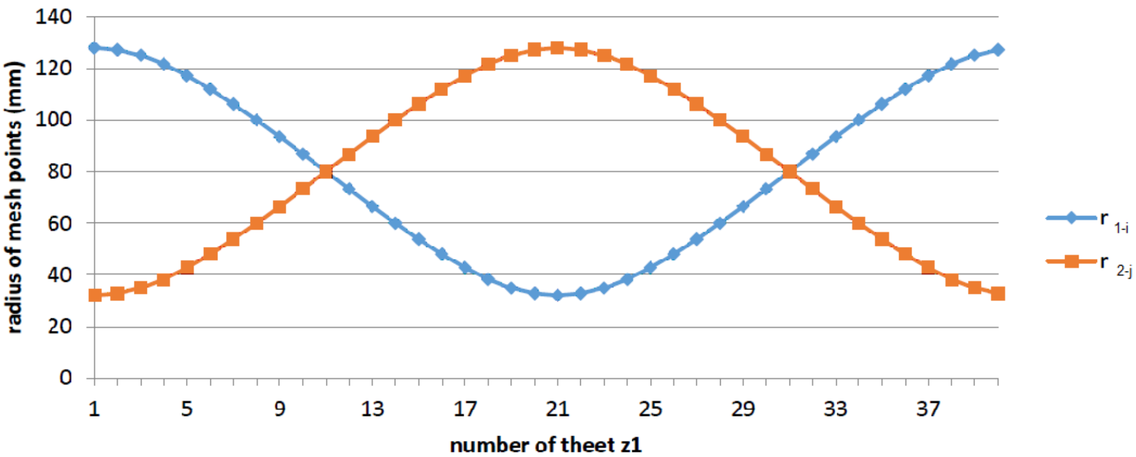

Fig. 7. Radius of the mesh points

The rotational speed of the drive wheel gear and the driven wheel gear is constant for standard spur gears. For the designed elliptical gearing with variable transmission, the angular velocity of the driven wheel is not constant but changed according to the continuous changing of the gear ratio. This is shown in Fig. 8, where the angular velocity is on the drive wheel ($\omega_1=100\ s^{-1}$) and the driven elliptical wheel (ω_{2i}).

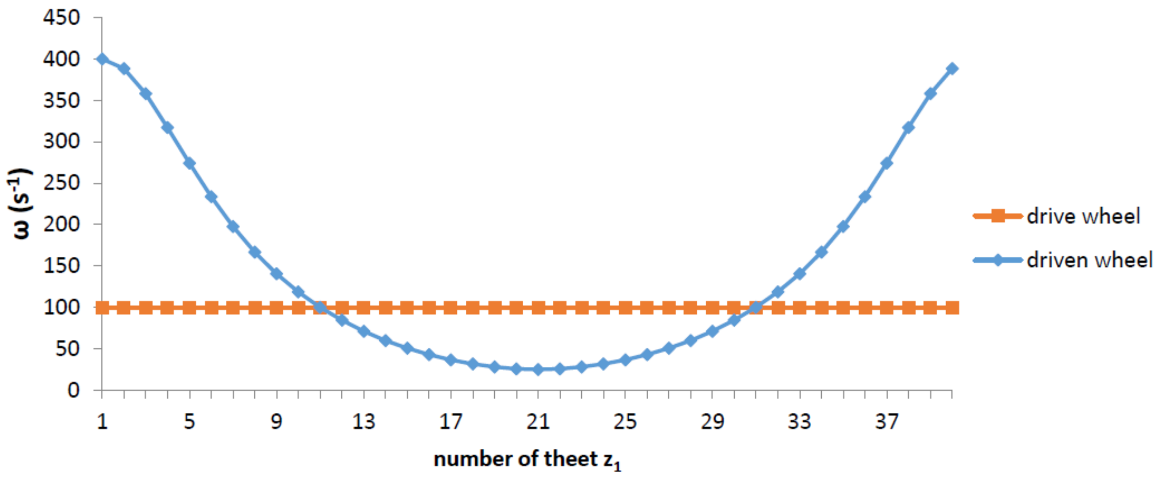

Fig. 8. Rotational speed in non-circular gearing

3.4. Modification the pitch ellipse when changing the gear ratio

For changes in gear ratio, for example, the variable transmission ratio, in the range $u=0.5$ to 2, the modification of the pitch ellipse is necessary. Fig. 9 presents the shape of the pitch ellipse for the non-circular gear, which is defined by the number of teeth $z_1=z_2=40$ and gearing module $m_n=4\ mm$, where the distance $a=160\ mm$ and the gear ratio is $u=0.5$ to 2.

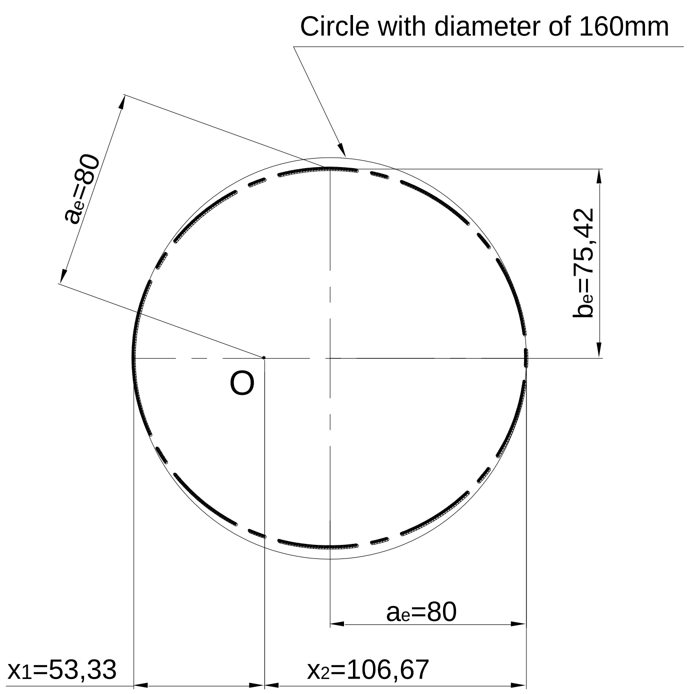

Fig. 9. Pitch ellipse for the gear ratio u=0.5 to 2

The position of ellipse focus is determined by considering the desired continuously changing transmission gear ratio. For the gear ratio in the range u=*0.5* to *2*, the position of the ellipse focal point (centre point *O* of rotation) is determined by the ratio lengths x_1: x_2, which are equal to 1:2. The second half-axis $b_e=75.42\ mm$ is determined by the distance from the focus point $a_e=80\ mm$ for the transmission ratio $u=1$. The position of the pitch ellipse focus is consistent with the variable gear ratio.

4. CONCLUSION

This article describes how to optimize the design of pitch curves of non-circular gears for given parameters. Non-circular gearing consists of two identical gear wheels. For a non-standard gearing, an eccentric elliptical gear drive with a continuously changing transmission gear ratio was applied. The kinematic properties of this gearing are different from the properties of standard circular gears, i.e., spur gears. Thus, the gear ratio changes over the duration of one revolution.

Non-circular gears synthesize the advantages of circular gears and cam mechanisms, as well as offer a combination of high output power and excellent accuracy with continuously variable transmission. Non-circular gears have been applied to construction machinery, machine tools, and the automotive, aerospace and other fields.

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