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DETERMINING THE PROPERTIES OF PNEUMATIC FLEXIBLE SHAFT COUPLINGS WITH WEDGE FLEXIBLE ELEMENTS

Summary. At the Department we deal with the development of pneumatic flexible shaft couplings, which in addition to other flexible couplings are able to change their torsional stiffness by adjusting the air pressure in their flexible elements. This article deals with the computation of static load characteristics of pneumatic flexible shaft coupling with wedge flexible elements. This coupling was developed to improve the properties of pneumatic flexible couplings, especially the nominal and maximal torque and maximum angle of distortion. Due to the reason that coupling with wedge elements isn't manufactured yet, we will use only a mathematic model of this coupling.
Key words. Pneumatic flexible shaft coupling, static and dynamic torsional stiffness, coefficient of viscous damping

OKREŚLENIE WŁAŚCIWOŚCI PNEUMATYCZNEGO SPRZĘGŁA ELASTYCZNEGO Z ELEMENTAMI KLINOWYMI

Streszczenie. W katedrze zajmujemy się badaniem i rozwojem elastycznych sprzęgieł pneumatycznych łączących wały, które w przeciwieństwie do innych sprzęgieł dodatkowo umożliwiają zmianę ich sztywności za pomocą zmiany ciśnienia medium gazowego w ich elastycznych elementach pneumatycznych. Niniejszy artykuł przedstawia obliczenia statycznej charakterystyki obciążeniowej elastycznego sprzęgła pneumatycznego, łączącego wały z elementami klinowymi. Sprzęgło to zostało zbudowane w celu poprawienia właściwości sprzęgieł pneumatycznych, przede wszystkim nominalnego i maksymalnego obciążającego momentu skrętnego, a także maksymalnego kąta skrętu. Ponieważ ten typ sprzęgła nie jest jeszcze produkowany, zastosujemy jego model matematyczny.
Słowa kluczowe. Pneumatyczne sprzęgło podatne, statyczna i dynamiczna sztywność skrętna, współczynnik tłumienia wiskotycznego

1. INTRODUCTION

Previously known flexible shaft couplings are manufactured with metal, rubber or plastic flexible elements. The most widely used flexible couplings in engineering are flexible shaft

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couplings with rubber flexible elements. Durability and hence the life-time of rubber flexible element is closely connected with the heating of the coupling and hence the heating of its flexible elements. Permanent heat causes progressive fatigue of flexible elements. With fatigue rubber materials lose its original dynamic properties.

The above disadvantages of the current flexible shaft couplings are removed and the requirement demanded for new types of couplings are fulfilled with pneumatic flexible shaft couplings with wedge flexible elements, namely pneumatic tuner of torsional vibration with wedge flexible elements, developed at our department. This type of flexible shaft couplings in addition to other flexible couplings are able to change their torsional stiffness and hence the dynamic properties of mechanical systems using this type of flexible couplings. This is able by adjusting the air pressure in their flexible elements.

Currently manufactured pneumatic flexible elements are designed for linear deformation. Wedge elements have an air bellow designed for use in pneumatic flexible coupling and the deformation on a circular arc trajectory. This allows us to use more flexible elements in pneumatic flexible coupling and achieve a greater twist angle.

The aim of the paper is to determine the static load characteristics of pneumatic flexible coupling with wedge elements by calculation.

2. PROPERTIES OF NEWLY DEVELOPED PNEUMATIC SHAFT COUPLING WITH WEDGE FLEXIBLE ELEMENTS

Pneumatic torsional oscillations tuner with wedge elastic elements (fig. 1) consists of driving hub (1) and driven hub (2) with the supporting surface (3) and (4), among which are air-spring units.

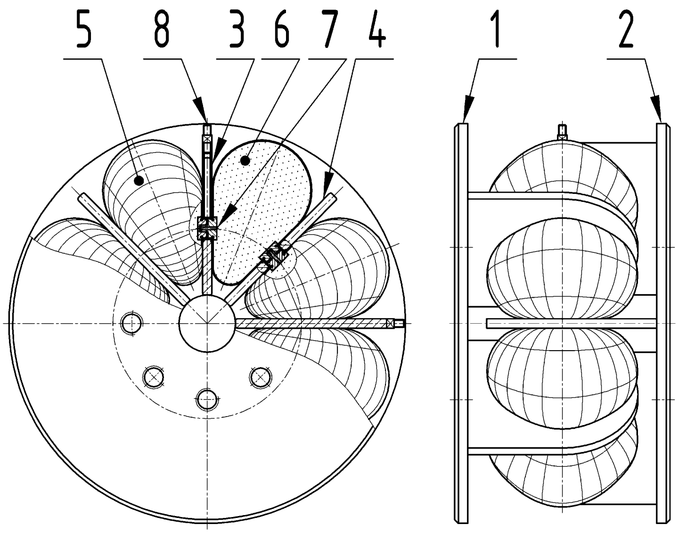


Fig. 1. Pneumatic tuner of torsional vibration with wedge flexible elements type 8 – 1/110 – T – C
Rys. 1.Sprzęgło elastyczne pneumatyczne z elementami klinowymi typu 8 – 1/110 – T – C

Each pneumatic flexible unit comprises of two flexible elements, namely a compressed flexible wedge element (5) and also extended wedge flexible element (6) Interconnection between wedge flexible elements (5) and (6) and thus between the compression spaces are provided by throttle openings (7). If compression space of coupling is filled with gaseous medium through valve (8) to a predetermined pressure, this keeps the driving hub (1) against the driven hub (2) in the basic position. Transmitted oscillating load torque causes deflection of the driving body (1) against the driven body (2). As a result, creates, as already mentioned, the compression of gaseous medium in compression chambers of wedge flexible elements (5) and (6) proportional to the load. Simultaneously the oscillating component of the torque load causes pulsing of the gaseous medium in the compression chamber of coupling, which forces a flow of medium through interconnecting throttle openings (7) proportional to oscillation.

The basic nature of pneumatic tuner’s design is that the loading torque is transferred from the driving hub to the driven hub by compression space, which consists of air-filled flexible pneumatic units [2].

3. MATHEMATICAL MODEL OF PNEUMATIC COUPLING WITH WEDGE FLEXIBLE ELEMENTS

Investigated pneumatic flexible shaft coupling with wedge elements is shown on fig. 2.

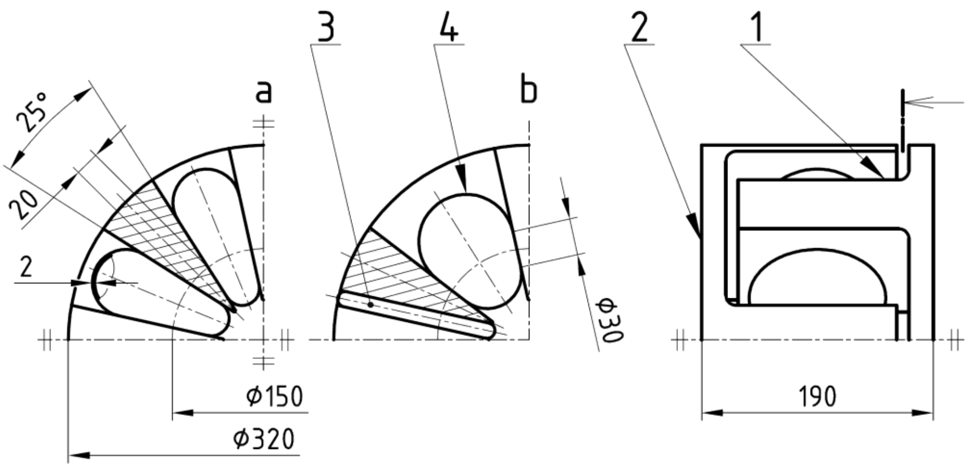


Fig. 2. Sprzęgło elastyczne pneumatyczne z elementami klinowymi typu 8 – 1/110 – T – C w pozycji neutralnej (a) i z maksymalnym kątem skrętu (b)

Rys. 2. Flexible shaft couplings with pneumatic wedge elements type 8 – 1/110 – T – C in neutral position (a) and by maximum twist angle (b)

Since that this type of flexible tuner isn’t currently manufactured, it was necessary to determine its basic characteristics theoretically based on a mathematical model. All dimensions necessary to calculate the static load characteristics are shown on fig. 2.

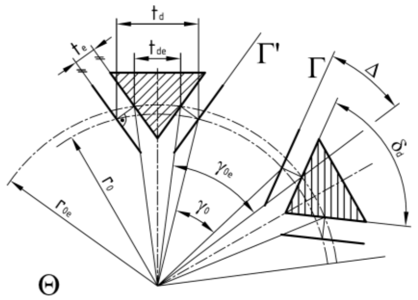
For static load characteristics computations the following conditions were considered:

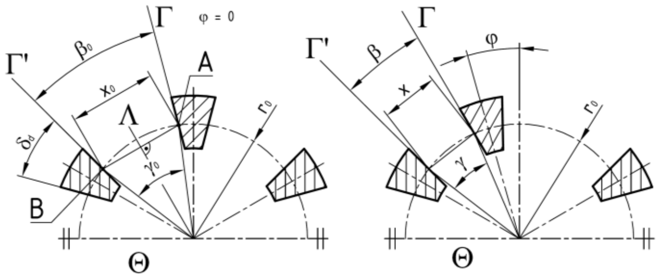
- volume of the interconnecting and filling tubes are neglected, as well as reduction of the bellows volume by the flange of element,
- we considered only the gas volume enclosed inside the air bellow of element,
- compression volumes of wedge elements are interconnected,
- neutral surface of the bellow lies in the middle of the tire‘s thickness,
- the length of meridial fibres of neutral surface was considered constant [2],
- the contact surface between elements and hubs is planar,
- in the part where flexible elements do not touch the supporting surfaces, meridial fibres of neutral surface are circular arcs [2], touching the equidistants of supporting surfaces,
- wedge elements has been designed so that contact surface between hub and maximally stretched element forms a circle with a diameter of *30 mm*,
- under static loading, the gas compresses and expands isothermally [2]
- equal absolute values of loading torque work and mechanical work of compressing air

4. GEOMETRY OF WEDGE PNEUMATIC FLEXIBLE ELEMENT

For the computation of static properties of pneumatic flexible coupling with wedge flexible elements it is necessary to know the geometric properties of wedge flexible elements bellow depending on twist angle [4].

On fig.3 is shown a scheme of couplings supporting surfaces in plane Θ, which is the elements plane of symmetry and it is perpendicular on the couplings rotation axis. Planes Γ and Γ' are planes which passes trough the bellows neutral surface in the contact area between the bellow and supporting surface. On fig.4 is shown the location of planes Γ and Γ' in neutral position and by twist angle φ. Plane Λ is the plane of symmetry between planes Γ and Γ'.





a) b)

Fig. 3. Determining the dimensions of neutral surface from external dimensions

Rys. 3. Określenie rozmiarów powierzchni neutralnej na podstawie powierzchni zewnętrznej

Fig. 4. Determining the dimensions of neutral surface, in neutral position (a) and by maximum twist angle (b)

Rys. 4. Określenie rozmiarów powierzchni neutralnej, w pozycji neutralnej (a) przy maksymalnym skręcie (b)

For known number of elements i, thickness of supporting surface t_{de}, pitch radius of elements r_0 and supporting surface angle δ_d we can compute angles Δ and β_0 in the neutral position of coupling:

$$\beta_0 = \frac{2\pi}{i} - \delta_d \qquad (1)$$

$$\Delta = \frac{\delta_d}{2} - arcsin\left(\frac{t_{de}}{2 \cdot r_{0e}}\right) \qquad (2)$$

Then for the known bellows external thickness t_e , t_d and r_0 for the neutral surface are:

$$t_d = t_{de} + 2 \cdot t_e \cdot cos\frac{\delta_d}{2} \qquad (3)$$

$$r_0 = \sqrt{(r_{0e} - t_e \cdot sin\,\Delta)^2 + (t_e \cdot cos\,\Delta)^2} \qquad (4)$$

Then we determine the angles γ_0 and γ_{0e}:

$$\gamma_0 = \frac{2\pi}{i} - 2 \cdot arcsin\left(\frac{t_d}{2 \cdot r_0}\right) \qquad (5)$$

$$\gamma_{0e} = \frac{2\pi}{i} - 2 \cdot arcsin\left(\frac{t_{de}}{2 \cdot r_{0e}}\right) \qquad (6)$$

For angles β and γ by twist angle φ applies:

$$\gamma = \gamma_0 - \varphi \qquad (7)$$

$$\beta = \beta_0 - \varphi \qquad (8)$$

Points *A*, *B* are the centres of elements flanges on the neutral surface of elements bellow. Their flowline is the axis of element bellows neutral surface. The distance *x* of points *A*, *B* is determined as:

$$x = 2 \cdot r_0 \cdot sin\frac{\gamma}{2} \tag{9}$$

The element bellows geometry is determined in the section of plane *Π*, which passes trough the flowline of points *A* and *B*. The location of plane *Π* is determined by angle ξ (fig. 5)

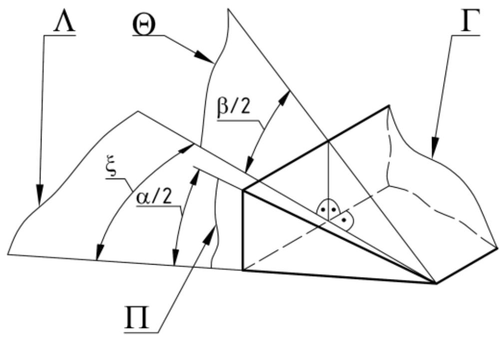
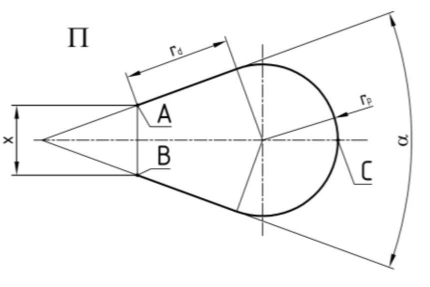


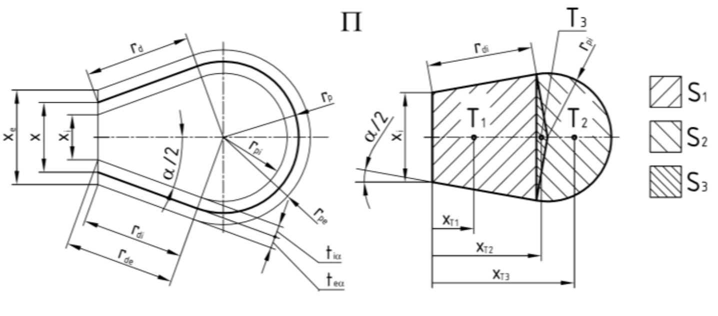
Fig. 5. Relative position of flexible elements bellows planes
Rys. 5. Wzajemne położenie powierzchni płaszcza elementu elastycznego

The angle between support surfaces α in plane *Π* is described by formula:

$$\alpha = 2 \cdot arctg\left[cos(\xi) \cdot tg\left(\frac{\beta}{2}\right)\right] \tag{10}$$

On fig. 6 is shown the form of meridial fibre of bellows neutral surface in plane *Π*, on fig. 7a is shown the bellow in section, on fig. 7b is shown the cross-sectional of area between one side of internal surface and the axis of element bellows neutral surface.





a) b)

Fig. 6. Meridial fibre of neutral surface in plane Π
Rys. 6. Włókno południkowe powierzchni neutralnej w płaszczyźnie Π

Fig. 7. Determining the characteristics of internal surface in plane Π, dimensional and angular (a) sectional (b)
Rys. 7. Określenie właściwości długości i kątów (a) przekrojów (b) powierzchni wewnętrznej w płaszczyźnie Π,

For the condition of meridial fibres constant length l_{vp} depending on twist angle φ then follows:

$$r_d = \frac{l_{vp} \cdot cos\left(\frac{\alpha}{2}\right) - \frac{\alpha+\pi}{2} \cdot x}{2 \cdot cos\left(\frac{\alpha}{2}\right) + (\alpha+\pi) \cdot sin\left(\frac{\alpha}{2}\right)} \qquad (11)$$

$$r_p = \frac{x}{2 \cdot cos\left(\frac{\alpha}{2}\right)} + r_d \cdot tg\left(\frac{\alpha}{2}\right) \qquad (12)$$

The main property of wedge elements is that they are specially designed for deformation on circular arc. Therefore the length of fibre $l_{vp} = A\widehat{C}B$ depends on the angle ξ. When designing the bellow we assume that bellow will touch the supporting surface in circular area with radius r_{dz} for maximally expanded element (by maximum twist $-\varphi_{max}$, where $x = x_{max}$):

$$r_{p(x_{max})} = \frac{x_{(-\varphi_{max})}}{2 \cdot cos\left(\frac{\alpha}{2}\right)} + r_{dz} \cdot tg\left(\frac{\alpha}{2}\right) \qquad (13)$$

$$l_{vp} = 2 \cdot r_{dz} + (\alpha+\pi) \cdot r_{p(x_{max})} \qquad (14)$$

The dependency of length l_{vp} on angle ξ for the studied coupling is on fig.8.

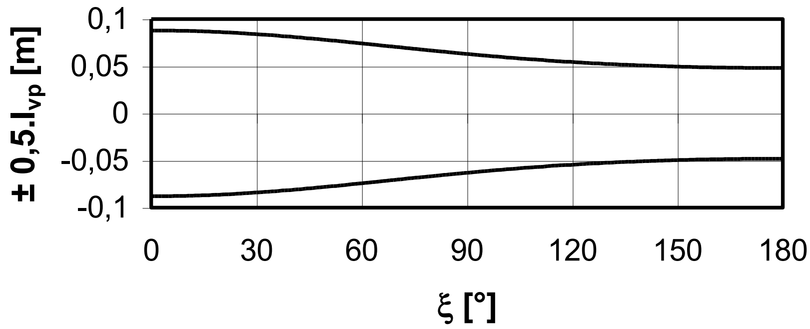


Fig. 8. Length of meridial fibre of neutral surface in plane Π
Rys. 8. Długość włókna południkowego powierzchni neutralnej w płaszczyźnie Π

For classic pneumatic elements (rotationally symmetric) l_{vp} will be constant all around the perimeter of element. For thus designed bellow it is necessary to check if radius r_p does not drop under minimum value by maximum twist angle (where ξ=0 or ξ=180°).

The internal and external distances of elements centres x_i and x_e we compute as:

$$x_i = x - 2 \cdot \frac{t_i}{cos\left(\frac{\beta}{2}\right)} \qquad (15)$$

$$x_e = x + 2 \cdot \frac{t_e}{cos\left(\frac{\beta}{2}\right)} \qquad (16)$$

As plane Π is perpendicular on plane Γ only for ξ = 0 and ξ = 180°, so the internal and external thickness of bellow (t_{ia} and t_{ea}) depends on the angle ξ (α):

$$t_{i\alpha} = \frac{x - x_i}{2} \cdot cos\left(\frac{\alpha}{2}\right) \qquad (17)$$

$$t_{e\alpha} = \frac{x_e - x}{2} \cdot cos\left(\frac{\alpha}{2}\right) \qquad (18)$$

The internal and external dimensions of bellow in plane Π we determine by formulas:

$$r_{pi} = r_p - t_{i\alpha} \qquad (19) \qquad\qquad r_{pe} = r_p + t_{e\alpha} \qquad (20)$$

$$r_{di} = r_d + t_{i\alpha} \cdot tg\left(\frac{\alpha}{2}\right) \qquad (21) \qquad\qquad r_{de} = r_d - t_{e\alpha} \cdot tg\left(\frac{\alpha}{2}\right) \qquad (22)$$

Then we compute the areas S_1 to S_3:

$$S_1 = \left[x_i + r_{di} \cdot sin\left(\frac{\alpha}{2}\right)\right] \cdot r_{di} \cdot cos\left(\frac{\alpha}{2}\right) \qquad (23) \qquad S_2 = \frac{1}{2} \cdot r_{pi} \cdot sin\left(\frac{\alpha}{2}\right) \cdot \left[x_i + 2 \cdot r_{di} \cdot sin\left(\frac{\alpha}{2}\right)\right] \qquad (24)$$

$$S_3 = \frac{1}{2} \cdot (\pi + \alpha) \cdot r_{pi}^2 \qquad (25)$$

and the locations of their centres of area x_{T1} to x_{T3}:

$$x_{T1} = \frac{3 \cdot x_i + 4 \cdot r_{di} \cdot sin\left(\frac{\alpha}{2}\right)}{6 \cdot \left[x_i + r_{di} \cdot sin\left(\frac{\alpha}{2}\right)\right]} \cdot r_{di} \cdot cos\left(\frac{\alpha}{2}\right) \qquad (26) \qquad x_{T2} = r_{di} \cdot cos\left(\frac{\alpha}{2}\right) + \frac{1}{3} \cdot r_{pi} \cdot sin\left(\frac{\alpha}{2}\right) \qquad (27)$$

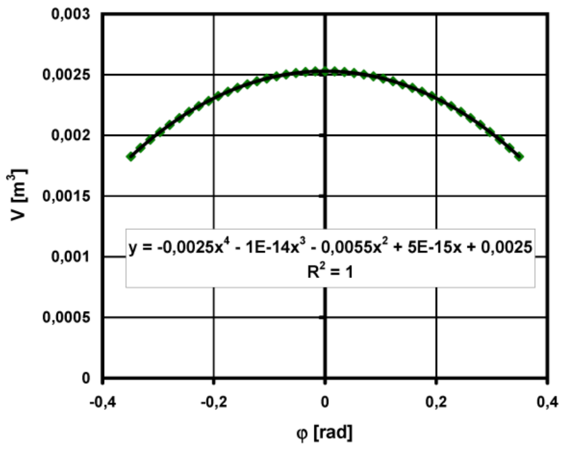
$$x_{T3} = r_{di} \cdot cos\left(\frac{\alpha}{2}\right) + r_{pi} \cdot sin\left(\frac{\alpha}{2}\right) + \frac{4}{3} \cdot \frac{r_{pi} \cdot sin\left(\frac{\alpha + \pi}{2}\right)}{\alpha + \pi} \qquad (28)$$

Now we can determine the internal volume of element V_e by using the static moments of area M_S:

$$M_S = \sum_{i=1}^{3} (x_{Ti} \cdot S_i) \qquad (29) \qquad\qquad V_e = \int_0^{2\pi} M_S \cdot d\xi \qquad (30)$$

5. COMPUTATION OF STATIC LOAD CHARACTERISTICS

For the computation of pneumatic couplings compression volume V we have to sum up the compression volumes of all elements and the volumes of filling and interconnecting tubes (neglected in this case). The compression volume V twist angle φ graph is shown on fig. 9.



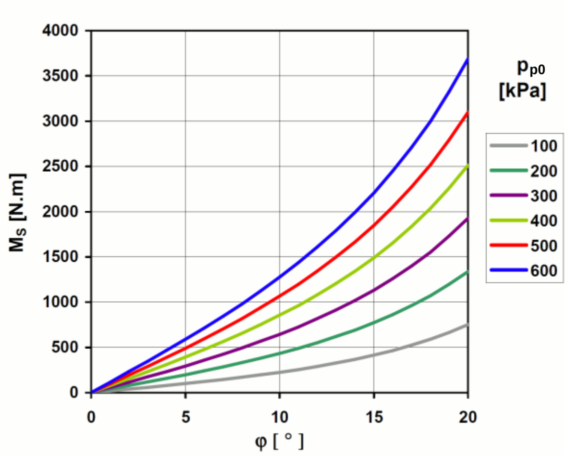


Fig. 9. Volume *V* twist angle graph
Rys. 9. Zależność objętości *V* od kąta skrętu

Fig. 10. Static load characteristics
Rys. 10. Statyczna charakterystyka obciążenia

Then it is necessary to know the dependency of couplings static moment of effective area $S_e.r$ on twist angle φ (*31*) and finally it is possible to define the static torque of coupling M_S (*32*) depending on the initial overpressure of gaseous media inside of elements for isothermic compression [4], [5]:

$$S_e \cdot r = -\frac{dV}{d\varphi} \qquad (31) \qquad M_S = \left[\left(p_{p0} + p_a\right) \cdot \frac{V_0}{V_0 - \int_0^{\varphi} S_e \cdot r \cdot d\varphi} - p_a \right] \cdot S_e \cdot r \qquad (32)$$

where:

p_{p0} – initial overpressure of gaseous media by couplings neutral position [Pa];
p_a – atmospheric pressure [Pa];
V_0 – compression volume by couplings neutral position [m^3].

Static load characteristics of the flexible coupling determined from the mathematical model of flexible couplings is shown on fig. 10. The static load characteristics is the starting point to determine other parameters of flexible coupling as static and dynamic torsional stiffness, nominal and maximal torque and other parameters [3]

6. CONCLUSION

The computation method described in this article allows to design the shape of wedge flexible elements tire based on selected couplings dimensions, and subsequently to determine the mechanical parameters of so designed coupling. Subject of further research will be proposal of such a mathematical model of the pneumatic elastic elements tire, which will consider not only planar contact surface.

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