

Miroslav VEREŠ<sup>1</sup>, Miroslav BOŠANSKÝ<sup>2</sup>, Milan RACKOV<sup>3</sup>

## POSSIBILITY OF THE HCR GEARING GEOMETRY OPTIMIZATION FROM PITTING DAMAGE POINT OF VIEW

**Summary.** The gearing contact strength which is manifested by pitting damage on its teeth flanks is one of the most important problems to need solve in the process of gearing design and calculation. According to nowadays valid standards such calculations can be solved with high level of reliability for all usual gearing types. Suitable calculations for strain and working life of HCR gears has not been sufficiently researched up-to-date. It has been identified that in HCR gears some different process of pitting birth arose during gears working process. Therefore it is actual to deal with optimization of HCR gearing just from pitting damage point of view.

**Keywords.** HCR gearing, contact ratio coefficient, pitting, optimization.

## MOŻLIWOŚCI OPTYMALIZACJI GEOMETRII UZĘBIENIA HCR POD KĄTEM JEGO USZKODZENIA PRZEZ PITTING

**Streszczenie.** Wytrzymałość stykowa uzębienia, która przejawia się uszkodzeniem przez pitting na bokach zębów, jest jednym z najważniejszych problemów, które należy rozwiązać w procesie projektowania oraz obliczeniach uzębienia. Zgodnie z obecnie obowiązującymi normami, takie obliczenia można wykonać z wysoką niezawodnością dla normalnych typów uzębienia. Odpowiednie obliczenia wytrzymałości oraz trwałości dla uzębienia HCR nie zostały dotychczas zbadane, ponieważ powstanie pittingu podczas eksploatacji uzębienia HCR przebiega nieco inaczej niż w profilach standardowych. Właśnie dlatego należy zajmować się optymalizacją uzębienia HCR pod tym kątem.

**Słowa kluczowe.** Uzębienie HCR, współczynnik długości trwania przyporu, optymalizacja.

### 1. INTRODUCTION

The HCR gearing is nowadays commonly used in the automotive gearboxes. Advantage of using such gears just in the transport vehicles transmissions is that this art of gearing has, with comparison to standard involute teeth profiles, low noise and also low load distribution along the path of contact during the whole teeth mesh. Moreover, in order to get a further

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<sup>1 2</sup> Slovak University of Technology, Faculty of Mechanical Engineering, Námestie slobody 17, 812 31 Bratislava, Slovak Republic, e-mail: miroslav.veres@stuba.sk, miroslav.bosansky@stuba.sk

<sup>3</sup> University of Novi Sady, Faculty of Technical Sciences, Trg D. Obradovića 6, 21 000 Novi Sad, Serbia, e-mail: eng.milan.rackov@gmail.com

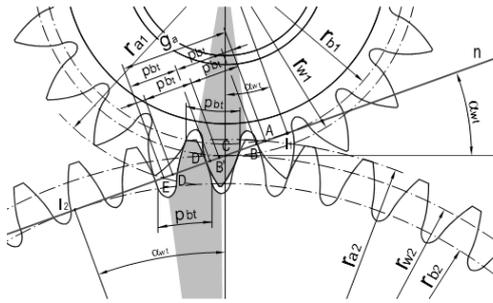


Fig. 1. Geometry of involute HCR gearing  
Rys. 1. Geometria ewolwentowego uzębienia HCR

the best behavior is obtained with an integer contact ratio, even though other specific non integer (rational) contact ratios can minimize the amplitude of some specific harmonics of the static transmission error. It is important to note that in Ref. [2] HCR gears were obtained by modifying the outside diameter; the other geometric parameters, e.g. the number of teeth, were left unchanged. The main indicator of HCR gearing, which differs from the commonly used standard involute profiles, is higher contact ratio, at least two pair of teeth in contact (see Fig. 1). The best conditions for decreasing in noise is caused by  $\epsilon_{\alpha}=2$  because there are always two

pairs of teeth in contact, which means when one pair of teeth go out from the contact, another pair of teeth is just coming in contact and applied force is considerably smaller and not varied since it is divided on two pairs of teeth. These favorable characteristics of the HCR gears should be reflected in increase of its resistance to contact fatigue damage - pitting, which is one of the main requirements for the design of the gears used e.g. in automotive transmissions. Creation of fatigue damage of the teeth flanks by pitting directly depends on the gear load. But from pitting damage point of view it is required to get contact ratio coefficient as high as possible. Workload at the three pair engagement divides between 2, respectively 3 pairs of teeth sides instead of one, respectively 2 pairs of sides (standard gearing), so its value decreases (Fig. 2).

## 2. PITTING IN THE HCR GEARING

According to current knowledge, creation and character of the pitting in the HCR gear is some different from the standard involute profiles. The main reasons are significantly higher slides, longer engagement and different load distribution between the teeth in gear. The area damaged by pitting is in the case of HCR gears more situated under the pitch cylinder, is more solid, the pitting is "more progressive", oval shaped and usually affects larger area than pitting generated in standard involute gears (Fig. 3). The shape and location of the pitting in the HCR profiles is clearly shown in the Fig. 4. Due to the larger

reduction of the vibration and teeth load, HCR gear profiles can be optimized. Sato et al. [1] found that HCR gears are less sensitive with respect to manufacturing errors. In particular, such kind of gears allows larger tolerance in the tip relief length. Moreover, they found that, in the absence of pressure angle error, the best contact ratio should be about 2. Kahraman and Blankenship [2] published an experimental work on HCR gear vibration; they found that

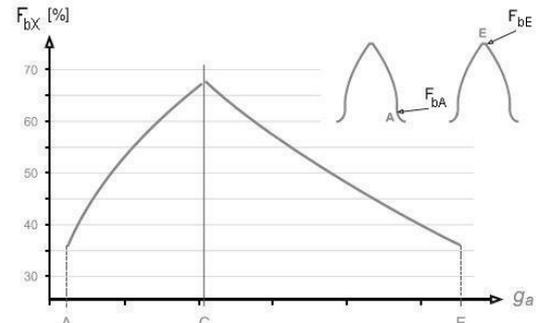


Fig. 2. Shape of the curves represents the load along the path of contact in one pair of HCR gearing teeth  
Rys. 2. Przebieg przenoszonego obciążenia jednej pary zębów uzębienia HCR wzdłuż odcinka

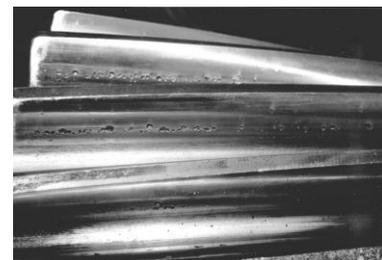


Fig. 3. Standard gear pitting damage

Rys. 3. Standardowe uszkodzenie uzębienia przez pitting

measurement delays at the beginning and the end of the engagement are the profiles significantly more prone to the creation of the temperature scuffing, which is clearly shown also in the Fig. 4.

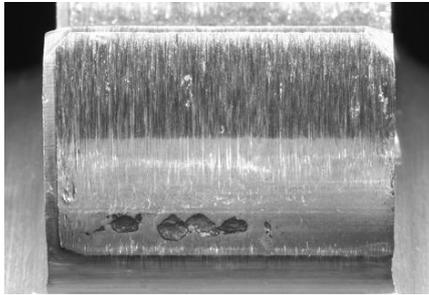


Fig. 4. Pitting in the HCR gearing  
Rys. 4. Pitting w uzębieniu HCR

Profit from a more favorable load distribution along the engagement is partially eliminated by less favorable course of the values of the reduced radius of curvature along the meshing (Fig. 5). Due to the longer contact line is its value in the peripheral areas lower, which of course leads to higher temperature scuffing and also to higher values of the Hertz pressure in contact area. It is clear shown on the Fig. 6 where are shapes of contact stress curves along the path of contact for HCR gearing with various geometrical parameters plotted [3]. It is clear to see that the contact stress of HCR gearing is strong depends on its geometry parameters. Therefore it is useful to optimize the HCR gearing just from this point of view.

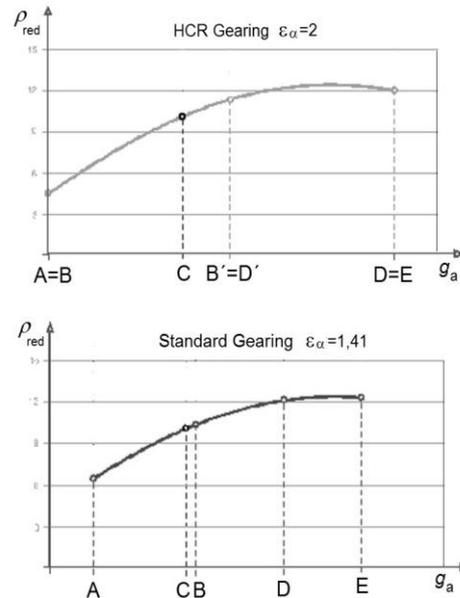


Fig. 5. Mean contact radius of curvature along the contact path  
Rys. 5. Zredukowany promień zakrzywienia wzdłuż odcinka przyporu

### 3. HCR GEARING GEOMETRY OPTIMIZATION AND ITS RESULTS

For Optimization of HCR geometry was used a new optimizing method called Generalized Particle Swarm Optimization Algorithm (GPSO). The GPSO algorithm was implemented in MATLAB. Using GPSO algorithm, solution of HCR value is obtained in a very short time, less than one second. This solution is very accurate, and it goes till 10<sup>-15</sup> accuracy. Calculated obtained results are presented further. Based on preliminary theoretical considerations optimization task with main objective functions

$$\sigma_{HX} = \sqrt{\frac{F b_{HX}}{\rho_r \pi b} \cdot \frac{1}{\left( \frac{1 - \frac{1}{\mu_1^2}}{E_1} + \frac{1 - \frac{1}{\mu_2^2}}{E_2} \right)}} = \min$$

$$\epsilon_\alpha = \frac{g_a}{p_{bt}} = \frac{\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a_w \cdot \sin \alpha_{wt}}{p_{bt} \cdot \cos \alpha_{wt}} = 2 \text{ or max}$$

was defined. It is clear, that above equations can be expressed as functions of pinion and wheel addendum coefficients and addendum modification factor of pinion ( $h_{a1}^*, h_{a2}^*, x_1$ ). Than as optimization parameters of this optimization task we can assume addendum  $h_{a1}$ ,  $h_{a2}$ , and addendum modifications coefficients  $x_1$  a  $x_2$ . Generally we can assume also  $\alpha_n$  but this is not advisable because this would mean decrease of value  $\alpha_n$ , what means also decrease of

gearing resistance against fatigue failure in tooth dedendum. For given value of centre distance  $a_w$  is known  $x_c$  and expressly defined relation between  $x_1$  and  $x_2$ . Then the objective functions can be express in the form  $\sigma_{HX} = f(h_{a1}^*, h_{a2}^*, x_1) = \min \varepsilon_\alpha = f(h_{a1}^*, h_{a2}^*, x_1) = 2$  or  $\varepsilon_\alpha = f(h_{a1}^*, h_{a2}^*, x_1) = \max$  with constrains of minimum top land width  $s_d \geq 0, 4m_n$  elimination of manufacturing and mesh interferences and slip ratio on begin and the end of mesh equalization. Results of such defined optimization task are shown on Fig. 7. All displayed points represent solutions convenient to mentioned constrains conditions.

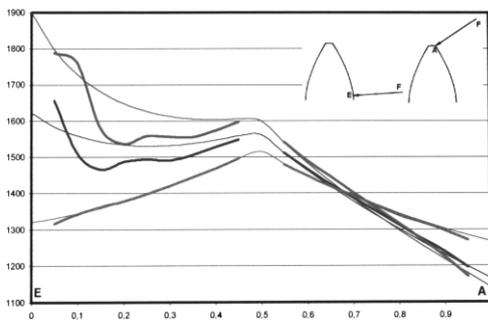


Fig. 6. Contact stress of HCR gears along the contact path for various geometry characteristics of gearing

Rys. 6. Napężenie stykowe w kołach HCR wzdłuż odcinka przyporu dla różnej geometrii uzębenia

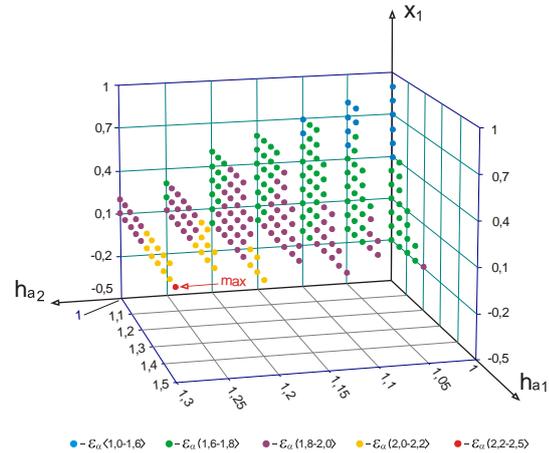


Fig. 7. Optimal values  $h_{a1}^*$ ,  $h_{a2}^*$ ,  $X_1$  for maximum contact load capacity depending on  $\varepsilon_\alpha$

Rys 7. Optymalne wartości  $h_{a1}^*$ ,  $h_{a2}^*$ ,  $X_1$  dla maksymalnej wytrzymałości stykowej w zależności od  $\varepsilon_\alpha$

#### 4. CONCLUSION

According to obtained results and their analysis it is possible to state that the optimization of HCR gearing geometry from pitting damage point of view is important task in the process of the transmissions design. There on the Fig. 8 are displayed results of such optimization task for concrete transmissions parameters ( $a_w=144\text{mm}$ ,  $z_1=21$ ,  $z_2=51$ ,  $m_n=4\text{mm}$ ). Optimization results that full meeting the requirements stated in the objective function and in the all constrains demonstrate that the whole process to find the optimal vector  $(h_{a1}^*, h_{a2}^*, x_1)$  is limited by low number of suitable combinations of the optimizing parameters.

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