

Aleksander KRÓL

STUDY OF AN OPTIMAL TRANSPORTATION NETWORK STRUCTURE STABILITY

Summary. There are various methods for the searching of the optimal transportation network structure for specified transportation needs. Apart from the principle, each method requires the construction of the network model with some simplifying assumptions and approximations, what leads to discrepancies with reality. Even larger uncertainties are caused by using of the data predicted for the transportation needs in the future. Obtaining of any single transportation network structure is therefore not yet a solution. Its behavior additionally should be examined for the changed data to ensure that it remains optimal both now and in the future covered by the forecast.

BADANIE STABILNOŚCI OPTYMALNEJ STRUKTURY SIECI TRANSPORTOWEJ

Streszczenie. Istnieją różne metody pozwalające na znalezienie struktury sieci transportowej optymalnej dla zadanych potrzeb transportowych. Niezależnie od zasady działania każda metoda wymaga budowy modelu sieci przy przyjęciu pewnych założeń upraszczających oraz dokonania przybliżeń, co prowadzi do rozbieżności z rzeczywistością. Jeszcze większe niepewności są skutkiem użycia danych prognozowanych dla potrzeb transportowych w przyszłości. Uzyskanie jakiegokolwiek pojedynczej struktury sieci transportowej nie jest zatem rozwiązaniem problemu. Należy dodatkowo zbadać jej zachowanie dla zmienionych danych, aby upewnić się, że pozostanie optymalną zarówno obecnie, jak i w objętej prognozą przyszłości.

1. INTRODUCTION

A transportation network is a layout of connections, in a region, between communities of people developed in the course of interaction of economic and social as well as natural environment factors. A transportation network serves the transport requirements of moving people and goods with different destination goals and relocation directions. The current form of transportation network is the result of long term development, which started with the first settlements in the region [9, 17]. When in the course of history modernisation and expansion was done, it was to relieve temporarily grown transport requirements, and the work was based on the current structure of the transportation network. These temporary requirements mostly arouse due to random factors, which as time went by ceased to prevail. In consequence the structure, as a rule, is not optimal.

At the moment, is observed a steep increase of communication needs associated with the intensification of economic growth, rising affluence of society and urbanization of new lands.

There is therefore a need to develop method enabling find the optimal structure of the transportation network in the area under consideration to meet communication demands. Taking into account different variants of future transportation needs it is possible to find various solutions. And the one, which offers fulfilling of this needs for long term and with acceptable costs should be chosen for the implementation [5].

1.1. The unstable systems

Creating of any model of any real system is the abstraction of some important system features and requires some simplifying assumptions and approximations. The design of the transportation network expansion is based on predicted future transportation needs. These long term forecast are usually marred by significant uncertainties. Inevitable imperfections of the model and the predictions uncertainty could lead to a solution, which is far from optimal. Additionally the whole data processing cannot be assumed to be linear. On the contrary, it is almost certain that some of the dependencies included in the model may be non-linear. There is therefore a risk of unstable behavior, which manifested high sensitivity solutions to small variations in input data. The assumption that a small disturbance of input data will always lead to the little disturbances of the results is therefore ineligible. It may happen, for a subset of input data space that a small value disturbance in the data would yield a solution of a completely different structure. This mechanism is shown schematically at Fig. 1.

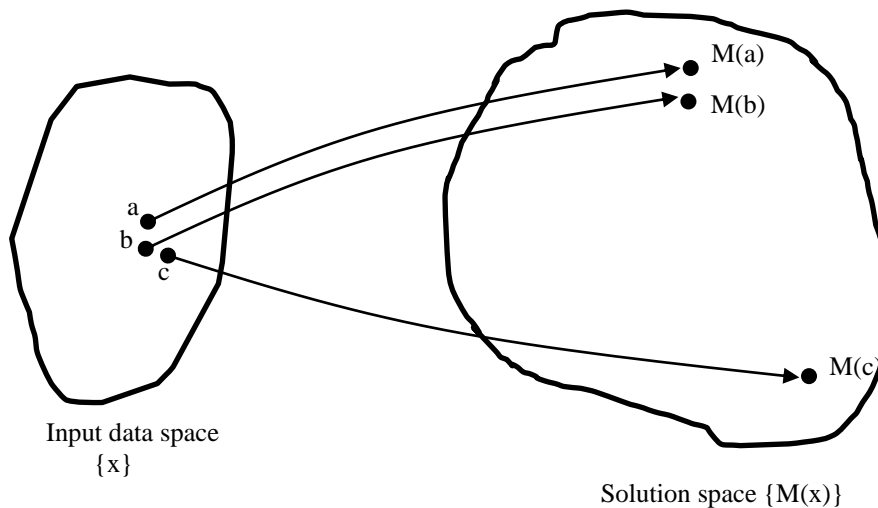


Fig. 1. An example of a solution very sensitive to small changes of input data

Rys. 1. Przykład rozwiązania czułego na małe zmiany danych wejściowych

This behavior resembles the behavior of chaotic dynamical systems [13], but due to the fact that the considered system is not described by differential equations it is only a superficial similarity. But advantages of some concepts introduced by the theory of deterministic chaos still can be taken. Useful among them is here, "basin" – a consistent subset of the input data space, which leads to almost identical solutions, or at least to the

solutions of similar structure. If the input data are located deep inside the basin of one of the solutions, this solution is resistant to the disturbing.

While the problems associated with the construction of the model can be greatly minimized through its improvements, even greater problems arising from the uncertainty of forecasting the of the transport needs intensity are practically unavoidable. It may therefore happen that the obtained structure of the transportation network is far from the correct one. And if the trend of the actual transportation needs in the future proves to be opposed to random disturbances of the data used for network designing the obtained structure would be quite inappropriate. This implies a need to investigate the stability of the transportation network structure under consideration and determine the corresponding basin in the space of input data. This approach requires the use of the method for the quick and easy generation of optimal transportation networks for different sets of input data.

2. TRANSPORTATION NETWORK STRUCTURE OPTIMISATION

The problem of stability of the solution is related to the task, not to a specific method of solving it. Nature of exploration of optimal transportation network structure problem excludes classical analytical methods. Data characterizing natural environment, localization of settlements and intensity of communication between them in general cannot be described analytically so it is natural to optimize such a system by artificial intelligence methods, particularly using genetic algorithms [6].

Optimization methods utilizing genetic algorithms mimic evolution processes in living nature [1]. In practice a genetic algorithm searching a scanty part of the solution space finds a solution as close as wished to optimal. Genetic algorithms surmount the fundamental problem of optimisation issues: it heads to a global maximum avoiding getting stuck in neighbourhoods of local maxima. It happens, because individuals, currently poorly fit, but potentially close to optimal solution, may enter consecutive optimisation stages (although with smaller probability). The measure of quality of the individual is called fitness function.

The crossover operation is one of the most important advantages of genetic algorithm in comparison with other methods using random searching e.g. simulated annealing: independent parts of poorly fit solutions could be near optimal and as a result of crossover could create an individual with high value of fitness function.

2.1. Assumptions used for defining the optimization model

Transportation network structure, in a given region, treated as an isolated object, is the subject of optimization. All interactions with neighbouring networks are modelled by introducing boundary nodes. Transportation network is represented as a graph, edges represent road connections and vertices represent intersections. During the optimization process the topology of the graph is modified – edges and vertices are removed or added, also the coordinates of vertices as well as the shape of edges may change. Additionally it is assumed that edges can have an extra attribute – class, which may also be optimized. Some of the vertices are fixed and are not subject to modifications – these represent actual towns.

Objective function minimized in the optimization process is the combined cost of constructing the network and of using it. Both constituents are examined over a long period of time in order to, after taking into account amortization, make comparisons plausible. Network construction cost is the cost of building all network connections, which is defined by the length of connections and unit costs of each connection dependent on its class and local area topographic details.

Network usage cost is the cost of using all connections, which is defined by the length of a connection, the number of vehicles travelling and unit costs of each trip dependent on the connection class. The loads of particular connections are derived on the basis of a traffic intensity matrix. Traffic intensity matrix represents volumes of traffic streams between network towns. Volume values are measured or estimated using various tools [14].

2.2. Genetic optimisation of transportation network

The principle of genetic algorithms imitates evolution processes of living organisms and is based on the following assumptions [1]:

- different solution versions compete with each other (individuals),
- structure of each individual is determined by a sequence of genes – genotype,
- genotype is subjected to random changes (mutations),
- randomly chosen individuals may exchange parts of their genotypes (crossover),
- fitness function being a measure of adaptation determines the probability of passing to the next generation (selection pressure),
- combining random mutations and crossover with selection pressure leads to optimal solution.

2.2.1. Genotype, mutation and crossover

The graph representing the transportation network is a complex structure and special mutation operators must be introduced:

- A) addition or deletion of connections,
- B) an existing connection could be linked with another node,
- C) addition or deletion of network nodes; nodes representing “towns” cannot be deleted,
- D) connection shape change,
- E) change of connection class.

Coordinates of points, which determine connection shape are real numbers. To obtain a mutation a random real number is added to the current value. The distribution of this correction is Gaussian with mean equal to 0 and standard deviation, which is a program parameter [10]. Typical value of standard deviation is about half of the extension of area being analysed. This approach promotes frequent, small corrections to point coordinates, but allows for rare, big changes too.

Crossover operators cannot function completely randomly and exchange any parts of genotypes, because this could give abnormal genotypes which do not represent networks. This problem was solved by introducing procedures searching, in the graph, for autonomous segments span between equivalent subsets of towns. If a set of such segments, for two individuals, is found their genotypes are exchanged. If a mutation or crossover causes an incoherence of the graph the individual is rejected.

2.2.2. Fitness function

The value of fitness function for an individual was defined as the inverse of the cost of constructing and of using a transportation network. Additionally an exponent, modifying the fitness function value is introduced. Changing this exponent value makes the shape of fitness function sharp or plain, and in this way chances of individuals, currently poorly fit can be adjusted. The default value of the exponent is selected $q = 2$.

$$f_f = \frac{1}{(K_B + K_U)^q} \quad , \quad (1)$$

where: K_B – constructing costs,
 K_U – usage costs.

Local differentiated building expenses depending on natural environment or infrastructure facilities can be represented as a map – “field of costs”. A suitable way to introduce this data is to use a grey scale bitmap. The values of pixels (g_i) along the connection represent local build cost; the white pixel) corresponds to no expense, the black one corresponds to the highest expense. The costs distribution could be additionally modified by the exponent (p , default $p = 1$). The connection class is included by suitable factor ($k_b[cl]$), and then the building cost of single connection can be expressed as [11]:

$$K_{Bi} = D_i k_b[cl] \sum_1^n \frac{1}{n} (255 - g_j)^p \quad , \quad (2)$$

where: D_i – length of connection,
 n – number of pixels along connection,
 g_j – value of pixel.

Much harder task is calculating the network usage costs by all vehicles basing on the traffic intensity matrix. While calculating the travel cost for single vehicle actual exploitation costs and cost equivalent of time wasted on travel should be considered. The cost of travel for a single vehicle thus depends on the current load. This relationship can be modeled in different ways, here it was assumed that this cost is proportional to the time taken on the passing of the part of the route. To calculate the travel time formula introduced by the U.S. Bureau of Public Roads is used [19]:

$$K_{Ui} = D_i k_u[cl] \left(1 + \alpha \left(\frac{O_i}{g[cl]} \right)^\beta \right) \quad , \quad (3)$$

where: D_i – length of the connection,
 $k_u[cl]$ – unit costs of the free driving traffic depending on connection class,
 O_i – current load [v/h],
 $g[cl]$ – capacity depending on connection class [v/h],
 α, β – calibrating parameters (0.15, 4).

Due to simplicity, at the present stage of development of the described model do not distinguish by type of vehicle. The average value of the unit cost at free driving has been estimated for each connection class [15]. Determination of the distribution of traffic flows between the alternative route is generally a complex optimization subtask [7, 8, 16].

It was assumed, that traffic flows are distributed in the transportation network in accordance with the second Wardrop's principle: each driver chooses the route to minimize his own cost.

To accelerate the calculations, instead of finding ways for each driver, an approximate procedure of dividing the total traffic volume only by a few dozen is used. At each step, starting from zero the intensity is increased by the same part, and then the currently cheapest route using Dijkstra's algorithm is looked for. The found routes are then charged with the current traffic fraction.

2.3. Uniqueness of the solution and the island genetic algorithm

The genetic algorithm is a nondeterministic procedure, so for a set of consecutive runs with the same data the different solutions could be obtained. There are two kinds of differences:

- small differences, mostly in detail of the connection shape, rarely in the connections class or network topology,
- a completely different network structure, but usually with similar fitness function value.

In the first case, the various approximations of the same global maximum are obtained, so only a small correction is needed to achieve the true optimum. The occurrence of the second case indicates that the problem has more than one solution. Possible solutions of the problem may have identical, although mostly have only close values of fitness function, which is a measure of their quality. In the context of approximations and simplifications made in the process of constructing a model, that were discussed in the introduction a small variations in the value of fitness function do not matter and all such solutions should be regarded as equivalent.

Selection of one of two or more solutions for the transportation network may be based on other considerations than those that are covered by the applied model. There is, however, equally important benefit of the greater number of equivalent or nearly equivalent solutions – study of the behavior of the entire family of solutions allows to identify those that are stable and remain unchanged over a wide range of variability of input data.

2.3.1. *The island genetic algorithm*

Meanwhile, classic genetic algorithm always tends toward only one of the solutions, and other variants are removed from the population. Such behavior is an indirect consequence of the building block hypothesis [1, 20]. There is therefore a need for such a modification of the genetic algorithm, to be able to simultaneously search for different variants of the solution. The simplest approach is to use an island model: the population is divided into fixed subpopulations, which evolve in principle separately. From time to time, however, the information is exchanged – it is allowed to crossover between individuals from different subpopulations [3, 4, 18].

The island model is not only simple paralleling of consecutive runs of classic algorithm. Through the information exchange between subpopulations best partial solutions are likely to spread throughout the whole population, however small probability of exchange guarantees the independence of the evolution and rather leads to different solutions.

In the shown version of island genetic algorithm uses a rigid, predetermined division into subpopulations, it requires an assumption about their maximum number.

3. RESULTS

The presented transportation network model is not yet advanced enough to apply it to the real area and the real traffic intensity data. In particular, the methods of obtaining and interpreting data for plotting maps of the transportation network construction costs should be refined [15]. So a number of simulations were carried out for the fictional configurations of cost maps, while changing in many ways transportation needs.

3.1. Some examples of the analysis of the stability of transportation network structures

The first examined structure was a simple transportation network linking the three cities located at the vertices of an equilateral triangle. The costs of road construction were the same throughout the whole area. The same traffic intensity value between each pair of cities was assumed. Only this parameter has been changing in this series of tests. Fig. 2 shows the structure obtained for various values of traffic intensity.

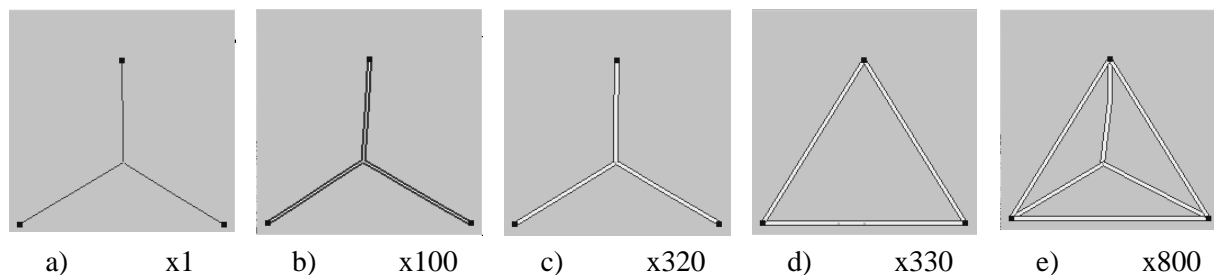


Fig. 2. Optimal transportation network structures obtained for various values of traffic intensity (given in relative units)

Rys. 2. Optymalne struktury sieci transportowej otrzymane dla różnych wartości natężenia ruchu (podanego w jednostkach względnych)

Even such a simple example shows that, at some range of input data can be a drastic change of transport network structure. Plan in the form of a star is optimal for small traffic flows in a very wide range of variability covering more than two orders of magnitude. In the transition region, at intensities of about 320 - 330 times the intensity of the initial, the optimal plan takes the form of a triangle. And such a construction plan is stable at any greater traffic intensity. The examined scope of traffic volumes was unrealistic wide, but the transition region is really narrow.

The model detailed assumptions imply successive changes of overall construction plan while traffic intensity keeps growing: five classes of road connections with the established maximum capacity were introduced, so the increase of the traffic demand causes connections duplication. In the first phase the star-shaped additional subnetwork is created (Fig. 2e), then

with further increase the shape of the newly established subnetwork becomes a second triangle (not shown).

Further examples of transport networks generated for a more realistic input data are not so easy to interpret, but also clearly show the presence of various overall construction plans depending on the assumed volume of traffic.

Fig. 3 shows the optimal structures of the transport network for the area of diverse construction costs, when the relative traffic volume widely changes from 1 to 30. As can be easily seen for each traffic volume was generated a pair of structures with almost the same quality. In each pair there are significant differences in how to communicate just the west city. Both the construction plans are almost stable throughout the all range of traffic volume variation. Only for the largest volume (x30), the structure which can be considered a hybrid of both plans to build was created.

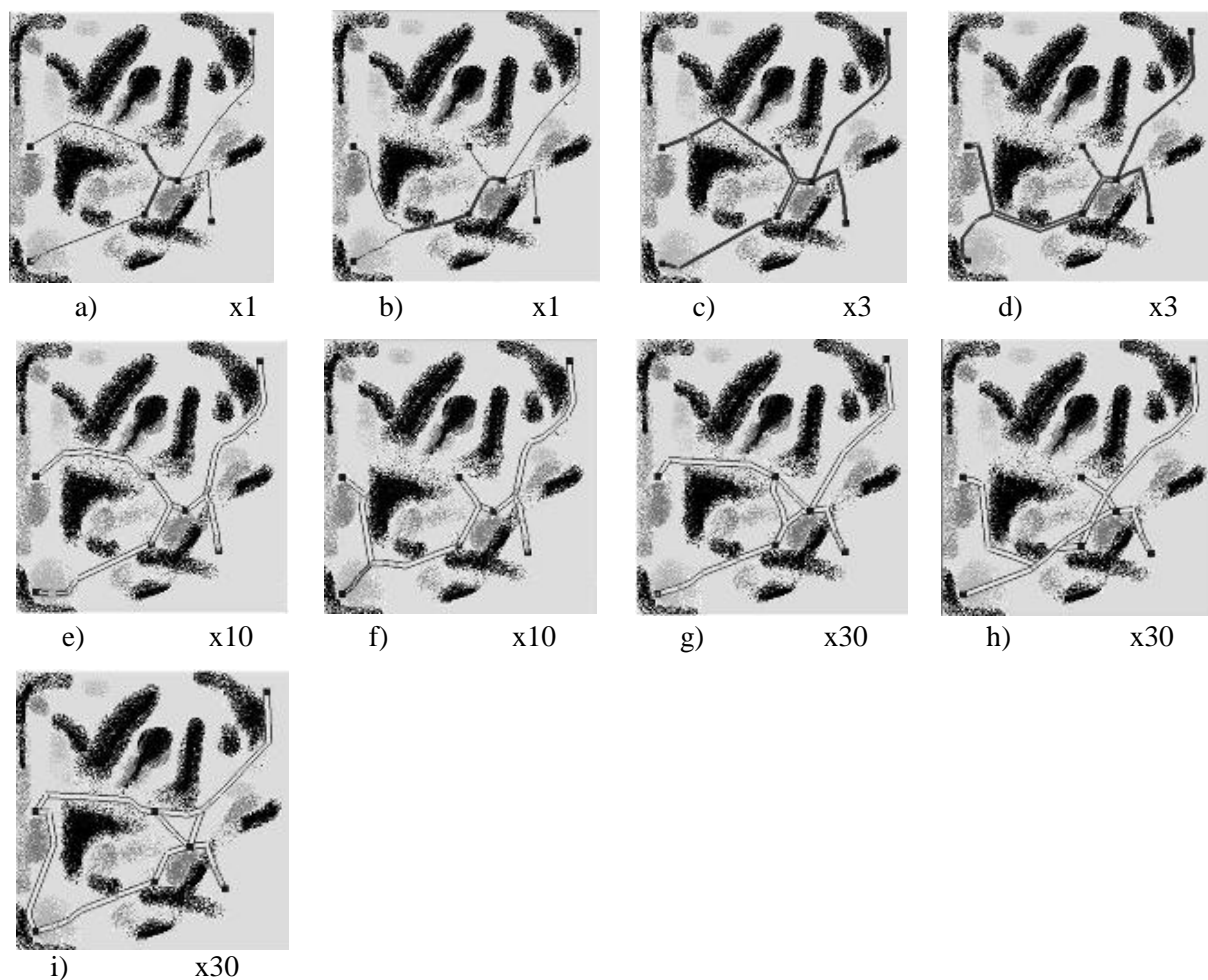


Fig. 3. Stable overall construction plans of the transport network in an area of diverse construction costs for different traffic volumes (given in relative units)

Rys. 3. Stabilne ogólne plany budowy sieci transportowych na obszarze o zróżnicowanych kosztach budowy dla różnych natężeń ruchu (podanych w jednostkach względnych)

4. CONCLUSION

The paper presents a method of finding the optimal transportation network for a given area with given transportation needs. The application of island genetic algorithm allows major peaks to be simultaneously found. These peaks represent different network structures fulfilling the transportation needs.

The described method makes possible to choose transportation network structure, which is resistant to inaccuracies in model construction. It is possible to include data from various, uncertain long-term forecasts. Analyzing obtained overall construction plans, one can find the most resistant to changes in input data ones. These solutions meet present requirements and will also give a favorable base for possible future expansion.

With such alternatives in the next step should be chosen one of them for implementation, and now may also be taken into account additional prerequisites, which failed to be included in the model.

Bibliography

1. Arabas J.: Wykłady z algorytmów ewolucyjnych, WNT, Warszawa 2004
2. Bertalanffy L.: Ogólna teoria systemów. Podstawy, rozwój, zastosowania, PWN, Warszawa 1984, s. 340.
3. Borovska P., Lazarova M.: Migration policies for island genetic models on multicomputer platform, in: Intelligent Data Acquisition and Advanced Computing Systems, Technology and Applications, IEEE, 2007, pp. 143 – 148.
4. Cantu-Paz. E.: Topologies, migration rates, and multi-population parallel genetic algorithms, 1999.
5. Drezner Z., Wesolowsky G. O.: Network design: selection and design of links and facility location, *Transportation Research Part A* 37, 2003, ss. 241–256.
6. Feng Ch., Lin J.: Using a genetic algorithm to generate alternative sketch maps for urban planning, *Computers Environment and Urban Systems* 23, 1999, pp. 91-108.
7. Gen M., Altıparmak F., Lin L.: A genetic algorithm for two-stage transportation problem using priority-based encoding, *OR Spectrum* 28, 2006, pp. 337–354.
8. Kim B. J., Kim W., Song B. H.: Sequencing and scheduling highway network expansion using a discrete network design model, *Ann Reg Sci* 42, 2008, pp. 621–642.
9. Kolars J., Malin H.: Population and accessibility: an analysis of Turkish railroads, *Geographical Rev.* 40/2, pp. 229-245.
10. Król A., Pamuła T.: A Genetic Algorithm For The Design Of An Optimal Transportation Network, proc. Int. Conf. „Telematics, Logistics and Transport Safety”, Silesian University of Technology, 2009, ISBN: 83-923434-2-5, pp. 75-81.
11. Król A., Pamuła T.: Application Of Genetic Algorithm For Designing A Transportation Network With Varying Construction Costs, proc. of Int. Conf. “AI-METH’2009”, Silesian University of Technology, Gliwice, 2009, pp. 179-187.
12. Morcousa G., Lounisb Z.: Maintenance optimization of infrastructure networks using genetic algorithms, *Automation in Construction* 14, 2005, pp. 129 – 142.
13. Ott E.: Chaos w układach dynamicznych, WNT, Warszawa, 1997.
14. Pamuła T., Król A.: Statystyczny model monitorowania ruchu drogowego, *Zeszyty Naukowe Politechniki Śląskiej, Seria TRANSPORT*, z. 6, 2009.

15. Pazik K.: praca magisterska: Wpływ kosztów budowy i kosztów użytkowania na modelowanie struktury sieci transportowej, Katowice, wrzesień 2010.
16. Pinninghoff M., Contreras R., Atkinson J.: Using Genetic Algorithms to Model Road Network, IEEE Computer Society, 2008.
17. Ratajczak M.: Modelowanie sieci transportowych, Wydawnictwo Naukowe UAM, Poznań, 1999.
18. Skolicki Z., Dejong K.: The influence of migration sizes and intervals on island models, in: GECCO '05: Proceedings of the 2005 conference on Genetic and evolutionary computation, New York, 2005, pp. 1295-1302.
19. Us Department Of Transportation: Traffic Assignment, Washington, 1973.
20. Król A.: Poszukiwanie alternatywnych struktur sieci transportowych za pomocą algorytmu genetycznego, Logistyka 6/2010, Poznań, 2010.