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TWO-LANE TRAFFIC FLOW MODEL FOR HIGHWAY NETWORKS

Summary. A discrete model to simulate multi-way traffic flow is introduced. The well known cellular automata Nagel-Schreckenberg model is extended by adding extra road lanes. New set of state rules is developed to provide lane change manoeuvre for vehicle overtaking and returning to lane designated for slower traffic. Results of numeric simulations are partially consistent with the so-called fundamental diagram (flow vs. density), as is observed in the real free-way traffic.

DWUPASMOWY MODEL RUCHU DROGOWEGO DLA SIECI AUTOSTRAD

Streszczenie. W Artykule przedstawiono dyskretny model ruchu drogowego. Znamy model Nagela-Schreckenberga oparty na automatach komórkowych, został rozszerzony o dodatkowe pasma ruchu. Opracowano nowy zestaw reguł zmiany stanów, umożliwiający manewr zmiany pasa ruchu: wyprzedzania oraz powrotu na pas przeznaczony do jazdy z mniejszą prędkością. Wyniki numerycznych symulacji są częściowo zgodne z podstawowym diagramem fundamentalnym (przepływ vs. gęstość), zależnością obserwowaną w ruchu rzeczywistym.

1. MOTIVATION – TRAFFIC FLOW QUALITY

Modelling traffic transport problem is very interesting and important for its dynamics and serious dramatic consequences in real life. The main goal of traffic flow control and road network design is to provide a qualitative description of traffic flow, especially to answer the question whether the traffic flow is equal to demand flow level over network in time and space [3]. The models, we examined, can be useful to provide proper tools to perform

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simulations for various scenario i.e. closed lane segment, lane speed limit, accidents, start-stop condition.

2. TRAFFIC FLOW PARAMETERS

In our research we focus on the main flow-density relationship which is the most important to reflect the traffic dynamics. This dependency is known as a Fundamental Diagram and is postulated as a certain function used for the approximation of observational data (see Fig. 1).
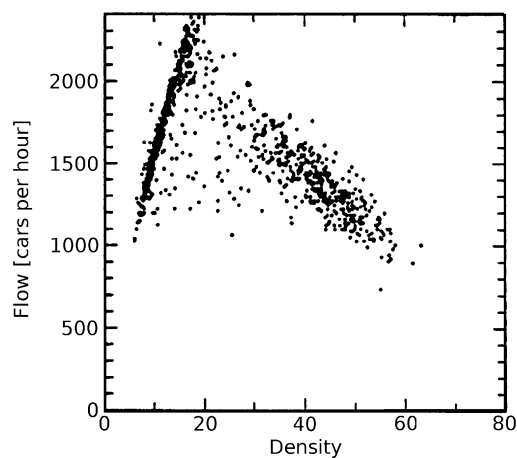

Fig. 1. Fundamental diagram. Real-life observation
Rys. 1. Zależność fundamentalna. Obserwacje rzeczywiste

Having the basic relationships between traffic flow, speed and density, special attention can now be directed toward the scale of view of traffic flow: macroscopic or microscopic.

3. CLASSICAL APPROACH

In the classical approach the traffic is mainly modelled as aggregated vehicle counts or traffic streams. The macroscopic treatment views the traffic as a fluid moving along a duct which is the road lane. The microscopic treatment considers the movement of individual vehicles as they interact with each other. In both approaches partial differential equations or delay differential equations are used.

Macroscopic approach

The macroscopic treatment views the traffic as a continuum similar to a fluid along a duct which is a highway. The discussed traffic along a reasonably crowded road has no appreciable gaps between individual vehicles. In such cases traffic may be viewed as continuum, and its characteristic corresponds to the physical characteristic of the imaging fluid. Macroscopic traffic flow models do not distinct vehicle-driver individual behaviour. This is the main issue we are concerned with, however, it is not observed during simulations [1].

Microscopic approach

Microscopic traffic flow models aim to describe the behaviour of individual vehicle-driver unit with respect to other vehicles in the traffic stream. Microscopic models are very suitable for the description of multiple user-class flow. However, the more realistic microscopic flow models are very complex. What is more, it has been argued that the assumptions underlying the equations describing the motion of each individual car are too difficult for validation, since the human behaviour in the real-life traffic is difficult to observe and to measure. This is unfortunate, since for reliable simulations, the microscopic parameters have to be just right. Consequently, many researchers and traffic flow management software use macroscopic traffic flow models instead [3].

Car following model

In car-following model (CFM) we postulate that an individual car's motion only depends on the car ahead [4]. Analysing driver behaviour, one can discover that human being has a time lag in reacting to any input stimulus. The human's decision of using break pedal has some delay [1]. This observation was a basis for the construction of theories, between them the simplest linear CFM equation that belongs to the class of second order, neutral type difference-differential equations, namely NDDEs [5].

4. DISCRETE MODELS

In the discrete models, the continuous quantities such as the position and the velocities of a car are approximated by (discontinuous) integer numbers. In our opinion, however, considered models have the ability to show phenomena observed both at macroscopic and microscopic levels.

We focus on the cellular automata approach (CA) instead of on the classical ones, i.e. fluid-dynamics approach [1], because of one important property of cellular automata, namely the lack of stability, i.e. very small changes in transition rules or states can have very dramatic consequences [6]. The biggest advantage of CA is that each cell of the automaton can reflect individual object characteristics. Since cellular automata are used widely in various disciplines, many definitions exist. We quote one of them [8].

Def. 1 *Cellular automata are dynamical systems in which space and time are discrete. A cellular automaton consists of a regular grid of cells, each of which can be in a finite number of k possible states, updated synchronously in discrete time steps according to local, identical interaction rules. The state of a cell is determined by the previous states of surrounding neighbourhood of the cell.*

Basic Nagel-Schreckenberg model

Nagel-Schreckenberg model known as the NaSch cellular automata model was originally defined by Nagel and Schreckenberg [11] in 1992. The model describes only one lane traffic with periodic boundary conditions. This means that the total number of vehicles is constant. The cell is empty or occupied by a vehicle. All cells are updated simultaneously. We use the

notation as follow: x_i denotes position of the vehicle, v_i is speed of the vehicle and g_i is a gap between leader and follower, and is defined as $g_i = x_{i+1} - x_i - 1$.

Then the set of rules is defined:

- acceleration of free vehicles: $v_i < v_{max} \wedge g_i > v_i + 1 \rightarrow v_{i+1} = v_i + 1$,
- slowing down due to other vehicles: $v_i > g_i - 1 \rightarrow v_i = g_i$,
- random breaking (noise): $v_i > 0 \rightarrow v_{i+1} = v_i + 1$ with probability p,
- vehicle motion: $x_{i+1} = x_i + v_i$.

The cell neighbourhood and the gap for NaSch simple rule are presented on the Fig. 2.

Fig. 2. Neighbourhood and gap of NaSch model
Rys. 2. Sąsiedztwo, oraz wolna strefa w modelu Nagela-Schreckeberga

Basic NaSch model assumes constant p for the third rule. It is insufficient for modelling some traffic flow phenomena i.e. start-stop state. Researchers who extend "random breaking" rules and proposed a velocity-depended randomisation (VDR) approach. It is a simple idea, namely the probability p is a function of the vehicle speed $p = p(v(t))$. In the simplest case the probability function is defined as follow:

$$p(v) = \begin{cases} p_0 & \text{for } v = 0 \\ p & \text{for } v > 0 \end{cases} \tag{3}$$

The first basic limitation of the NaSch model is that all drivers behave in the same manner. One of possible solutions of taking into taking this into account could be by assigning to different vehicles different maximum speeds. Moreover, the tolerated gap between vehicle could be driver dependent or could be a function of the speed.

The second weakness of the model is the deceleration rule (namely breaking) since in the present form it obeys a process under non-physical conditions: in some states vehicles reduce speed from v_{max} to 0 in one iteration.

5. MULTILANE DISCRETE TRAFFIC FLOW MODEL

Presented in this section approach is based on CA. First we define rectangular periodic lattice. Let α denote a periodic net, where each node contains single cell c.

$$\alpha = \{(i, j) \mid i, j \in N, 0 \le i \le n, 0 \le j \le m\} \tag{4}$$

Vehicles move on a two dimensional discrete space of the j-th cell located on the i-th lane. Every cell can either be empty or be occupied by one vehicle with the velocity $V \in (0, 1, ..., V_{max})$. Time evolves in the synchronous manner, at each discrete time step the arrangement of N cars is updated in parallel, according to a set of rules. In the multilane case,

we simply take a single lane and place each one alongside the other. We consider a two-lane model with periodic boundary conditions, where additional rules defining the exchange of vehicles between the lanes are introduced. It is clear that this extension can be made without changing the basic properties of the single-lane model.

For our model, we adapt the NaSch set of rules to provide vehicles' movement. We intentionally, have not included "random breaking" rule. Such model is known as the deterministic NaSch traffic flow model [7]. Since we are now using a deterministic, reversible and finite CA model with periodic boundaries, the corresponding traffic system is periodic in its system states.

$$V_{i,j}^{t+1} = \min\left(D_{i,j}^{+,t}, V_{\max i,j}, V_{i,j}^{t} + 1\right)$$
$$P_{i,j}^{t+1} = P_{i,j}^{t} + V_{i,j}^{t+1} \tag{5}$$

5.1. Extension of the set of rules

After many numerical experiments, we have simplified our model and introduced *no-accident condition*. It means the set of rules have to preserve against situations, where more than one car could occupy the same cell. We realise this approach and it helps us to define some generic rules; further research is under development.

Overtaking manoeuvre uses an extended neighbourhood and covers sites behind and ahead the vehicle, on the both lanes. We assume that the driver only detects the space occupancy of his neighbourhood. The speed of the other vehicle on the highway remains unknown for him. In consequence, some other strong assumptions have to be made to assert *no-accident condition*. We require the empty neighbourhood behind the car to ensure that only one car overtakes the considered cell. The need of the empty neighbourhood behind the car on the left lane protects against collisions with the vehicles that drive along the adjoining lane. All required conditions are below:

$$V_{0,j}^{t} < V_{\max i,j} \wedge D_{0,j}^{-,t} \geq D_{\max i,j} \wedge D_{1,j}^{-,t} \geq D_{\max i,j} \wedge D_{1,j}^{+,t} \geq D_{i,j}^{t} \rightarrow L_{1,j+(D-1)}^{t+1} = V_{0,j}^{t} + 1 \tag{6}$$

where:
− $D_{\max i,j}$ is distance cover by a vehicle at maximum speed (per one iteration),
− $D_{i,j}^{t}$ is distance cover by a vehicle at spot speed,
− $L_{1,j+(D-1)}^{t+1}$ is the value of cell at the relative position $j+D-1$, at the next time step.
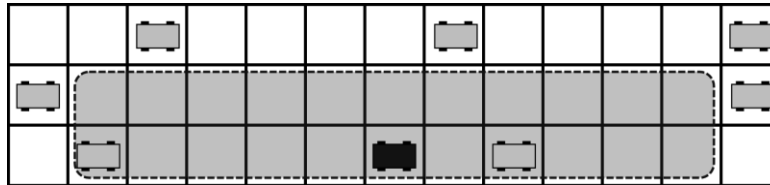

Fig. 3. Extended neighbourhood for overtake manoeuvre
Rys. 3. Rozszerzone sąsiedztwo dla manewru wyprzedzania

Returning manoeuvre satisfies requirement that left lane should be mainly used for the overtaking purpose. The rule is similar to that one used in the overtaking manoeuvre in [12].

$$D_{0,j}^{-,t} \geq D_{\max i,j} \wedge D_{1,j}^{-,t} \geq D_{\max i,j} \wedge D_{0,j}^{+,t} \geq D \to L_{1,j+(D-1)}^{t+1} = V_{0,j}^{t} \qquad (7)$$

For multilane model, extended notation is introduced. The lower indices i and j denote lane and vehicle (or empty cell) respectively. The upper index t defines the point in the discrete time domain. The upper symbol $^{+}$ or $^{-}$ denotes direction to the closest neighbouring vehicle. Thus, $D_{i,j}^{+,t}$ is distance to the nearest leader at the i-th lane, at relative position j, and $D_{i,j}^{-,t}$ is distance to the nearest follower. To reflect drivers various maximal (or preferable) speed we introduce constant $V_{\max i,j}$ for each of them.

5.2. Corrected lane-change rules

However, in our previous work [15] we have shown, that the introduced overtake manoeuvre should be preserved and the lane changing occurs sporadically even at a low density. Such rules are useless – our multi lane model behaves like multi independent lanes. In consequence, we have developed another approach, where rules are less preserved and still satisfy no-accident condition. Now, the new overtake manoeuvre for car on the right lane, at position $L_{0,j}$, is defined with the condition:

$$G_{1,j}^{-,t} \geq V_{\max i,j} \wedge V_{1,j}^{t} + 1 \wedge G_{1,j}^{+,t} \geq V_{0,j}^{t} - 1 \to L_{1,j+(V_{0,j}^{t}-1)}^{t+1} = V_{0,j}^{t} \qquad (8)$$

where:
-   $G_{0,j}^{-,t}$ is the gap to the nearest follower at the right lane, at the relative position j,
-   $G_{1,j}^{+,t}$ is the gap to the nearest leader at the left lane, at the relative position j,
-   $L_{1,j+(V_{0,j}^{t}-1)}^{t+1}$ is the value of cell at the relative position $j+\left(V_{0,j}^{t}-1\right)$.
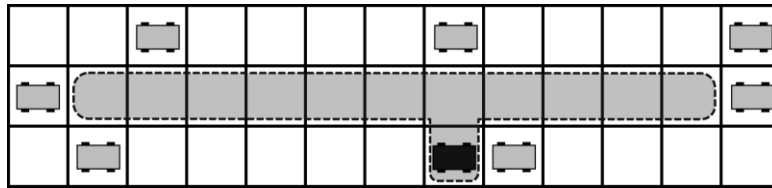

Fig. 4. Neighbourhood for overtake manoeuvre
Rys. 4. Zdefiniowane sąsiedztwo dla manewru wyprzedzania

We have used here other notation, which is more common in used. The distance D and the gap G, are equivalent. Such a rule works on smaller neighbourhoods (see Fig. 4. $V_{\max} = 10, V_{0,j} = 5$), and in consequence a smaller set of empty cells is required to process any lane change manoeuvre. Lane back makeover is symmetrical to overtake manoeuvre.

The defined lane change rule's neighbourhood has important drawback, it is possible that more than one vehicle will translate to the same cell (same position), In such case, in the implementation of the algorithm a list of vehicles that share the same cell is produced and then one car is selected (randomly) to occupy that cell. Then the NaSch rule set is applied to remaining vehicles. Fortunately, in our simulations the random choice in deterministic rules scheme occurs rarely and its effect can be neglected.

The new exchanged rules are defined by the following two criteria: first, a vehicle needs an incentive to change a lane; second, a lane change is only possible if some safety constraints are fulfilled. Rules are processed in the following priority order:

returning manoeuvre →overtaking manoeuvre →moving (NaSch rules' set)

However, only one single rule is applied per intermediate iteration. Main iteration I consists of three independent sub-iterations. Each sub-iteration processes one rule in a priority order. In the most situations vehicles just keep moving on the same lane. The rules are formulated under the following conditions:

– *returning manoeuvre* for each vehicle on lanes except for the first lane,
– *overtake manoeuvre* for each vehicle on the lanes except for the last left handed lane. The rule is applied if a car has no possibilities to develop its maximal preferable speed:

$$V_{0,j} < V_{\max} \wedge G_{0,j}^+ < V_{0,j} + 1,$$

– *moving manoeuvre* for each vehicle, on every lane, except for cars that have changed lane recently.

The above set of rules is minimal in the sense that they lead to a realistic behaviour and to the so-called fundamental diagram, i.e. the relation between the flow and the density is reproduced correctly. Unfortunately, some phenomena like spontaneous jamming will not occur in such system. One of solutions to perform could be that one in which more realistic simulation appears; the use of deterministic cellular automata model with stochastic boundary conditions [14] or with the open boundary conditions.

6. SIMULATIONS

We have assumed that the mean acceleration of passenger cars from 0 to 100 km/h is 12 second. The vehicle length is an approximation of passenger car size and equal to 4,5 meters. The further calculations are trivial, their main parameters are in Tab. 1. For automata size of $L = 1000[cells]$ the road length is 4,26 km. The 1000 iterations lasts about half an hour time period. The time step, one single iteration, ensures a sufficient time to perform the lane change manoeuvre.

Table 1

Model discretisation parameters

Parameters	Cellular Automata	Real environment
Car length	1 [cell]	4,5 [m]
Lane length	1000 [cell]	4,5 [km]
Car speed	(0,20) [cell/iter]	[8,215, 164,3] [km/h]
Car acceleration	-	2,315[m/s^2]
Time laps	1 [iter]	1,972 [s]

Beginning conditions

Beginning (Initiation) random condition was used in all processed simulations. The V_{max} (preferable speed) of each vehicle was constant and drawn from the uniform or arbitrary speed distribution. The one-modal speed distribution is the most common among drivers of passenger cars. The arbitrary bimodal distribution was used to reflect situation on the highway, where two sorts of vehicles dominate: passenger cars and trucks. The initial speed of car $V_{i,j}$ is drawn from range $\langle 0, V_{max} \rangle$. Finally, all vehicles are located randomly on both lanes. Density distribution over highway in time $t = 0$ is quasi uniform.

Sensitivity analysis

Numbers of numerical experiment have been performed to analyse the sensitivity of the model to the cellular automata length and to the number of iterations. The plots (Fig. 5.) show that the parameter flow converges very fast and, moreover, the chosen length L=1000 [cell] and the time I=1000 [iteration] are quite enough to obtain a stable solution.
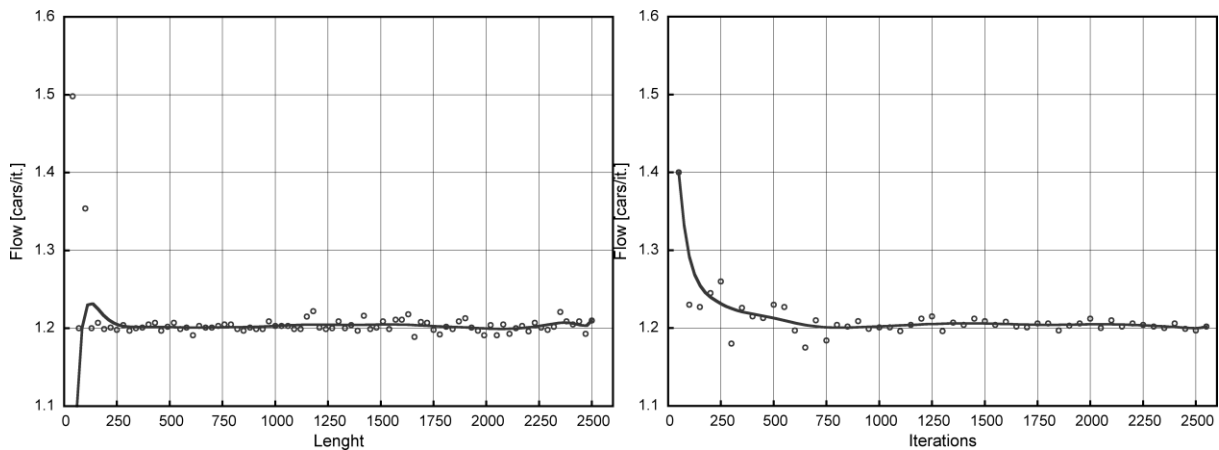

Fig. 5. Sensitivity analysis. Length of the automata, number of iterations
Rys. 5. Analiza czułości. Długość automatu, liczba iteracji

Please note that on all plots the approximation curves are only for visualization purpose. Default natural cubic spline approach has been used.

Results

As we have mentioned previously, the results are obtained from simulations on the lattice of 2 * 1000 sites with random initial configurations of vehicles. The population of N cars were randomly distributed in on both lanes around the complete loop with initial speeds sampled from $\langle 0, V_{max} \rangle$. The sensitivity analysis was done. The size of automata and number of iterations equal to 1000 is sufficient for the system to reach a stationary states. At this point of our research, we investigate some relations between preferred speed distribution and extreme points on the fundamental diagram (Fig.6.). At the higher density, the flow is stabilized and does not depend on driver comfortable speed preferences. The interesting

phenomenon is a very similar density-flow dependency for the constant preferable speed $V_{max} = 1$ and that drawn from the uniform distribution of $\langle 1,10 \rangle$.

The explanation is: the "idler drivers" slows down the traffic flow and the occurrence of overtake manoeuvre is still insufficient for other cars to develop a higher speed. We discovered, in bimodal arbitrary preferable speed distribution, that the fundamental diagram has some suspicious features. The flow at the low density is unpredictable. This effect is to investigate in further researches.
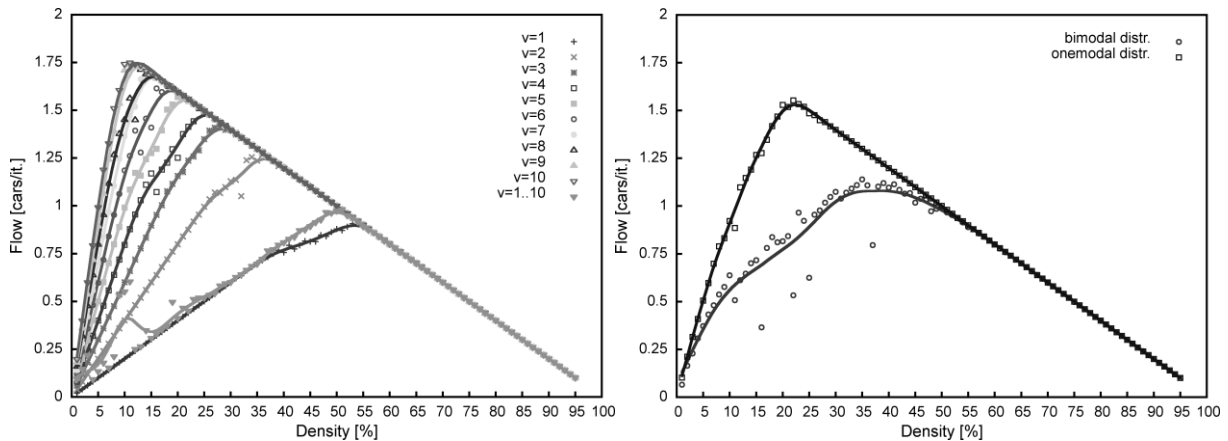

Fig. 6. Fundamental diagram. Uniform preferable speed distribution, one and bimodal speed distribution
Rys. 6. Diagram fundamentalny. Rozkład maksymalnej preferowanej prędkości: jednostajny, jedno
      i dwumodalny

We also investigated some aspect of the overtake manoeuvre occurrence. The relation between the overtakes manoeuvre versus the density and a preferable maximal speed distribution is highly non-linear. The number of lane change events varies spontaneously. Our hypothesis is that under some conditions a complex two-lane traffic flow model behaves like the Wolfram class 4 automata. What is more, the neglected influence of random process in the lane change rule may be false. These observations should be verified in further researches.

8. CONCLUSION

The proposed model is based on the Nagel-Schreckenberg cellular automata model without VDR. The solution of the highway traffic dynamics partially agrees with the real-life traffic. The lack of some stochastic noise influences the model behaviour. Some class of phenomena – spontaneous, unstable state, i.e. jam creation, kinematics waves, will not be reproduced in the strictly deterministic CA model. On the other hand, the new promising lane change manoeuvre set of rules were introduced. Presented algorithm fulfils *no-accidents* requirement and makes driver behaviour less preservative in comparison to the algorithms proposed in our previous works. We have begun to verify how the lane back and overtake manoeuvres influence the fundamental diagram, Unfortunately, for higher density we have observed, there are no car exchanging between lanes. Cars are moving along the same lane. Two-lanes traffic flow model behaves rather like two independent one-lane model. In the near future, we are

going to pay more attention to the set of rules that govern lane changing manoeuvres and to focus on unstable behaviour of the system at the low density.

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