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A STATIC CALIBRATION OF MEMS 3-AXIS ACCELEROMETER USING A GENETIC ALGORITHM

Summary. In this paper, a procedure for MEMS accelerometer static calibration using a genetic algorithm, considering non-orthogonality was presented. The results of simulations and real accelerometer calibration are obtained showing high accuracy of parameters estimation.
Keywords: MEMS accelerometers, calibration, bias, genetic algorithm

1. INTRODUCTION

Various microelectromechanical systems (MEMS) are used in different devices. Some of the most used are MEMS accelerometers. There are many areas of MEMS accelerometers application, such as location of objects, tracking of human movement, control of mirrors in cameras, etc. [2,3,5,11]. They are small, lightweight and provide digital outputs, but possess relatively large errors. Different ways of accelerometer calibration are used to improve accuracy.

The most important step for accuracy improvement is the static calibration of the accelerometer. It must be done separately for each particular accelerometer. There are two basic types of models of accelerometer static errors. One includes only biases and scale factors, which means 6 parameters [4,7,8]. The other models include terms for non-

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orthogonality between accelerometer axes [9,10]. It is more complex to estimate nine or more parameters, but in this way, a higher accuracy is achieved. Various methods for estimation are used. Some of them require special equipment for the perfect orientation of accelerometers [1,6,12,14], but those that do not require such are more useful [4,7-9].

Usually, algorithms of static calibration minimise the fitness function. Some of them used linearisation of the function [1,4,7], but they are sensitive to the presence of random. A lot of studies of algorithms without linearisation have been published [8,9,13,14].

Using a genetic algorithm for static accelerometer calibration is presented in this paper. A mathematical model of measurement is used to define the fitness function. The results of simulations and of calibration of real MEMS accelerometer are shown.

2. CALIBRATION ALGORITHM

Models of accelerometer measurement with nine parameters are shown in [9,10]. A slightly modified model is used in this paper:

$$\mathbf{a}^O = \mathbf{T} * \mathbf{K} * \left(\mathbf{a}^M - \mathbf{b} - \mathbf{n}\right) \tag{1}$$

where:

$\mathbf{a}^O$ is the acceleration vector in the accelerometer orthogonal frame (AOF)

$\mathbf{a}^M$ is the measured acceleration vector in the accelerometer body frame (ABF)

$\mathbf{b}$ is the vector of biases

$\mathbf{n}$ is the vector of the measurement noise, which is vector of white Gaussian noises;

$\mathbf{T} = \begin{bmatrix} 1 & \alpha_{yz} & \alpha_{zy} \\ 0 & 1 & \alpha_{zx} \\ 0 & 0 & 1 \end{bmatrix}$ is the transition matrix from ABF to AOF

$\mathbf{K} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix}$ is the matrix of scale factors

Averaging of measurements in time is done to exclude random errors:

$$\bar{\mathbf{a}}^O = \mathbf{T} * \mathbf{K} * \left(\bar{\mathbf{a}}^M - \mathbf{b}\right) \tag{2}$$

where: $\bar{\mathbf{a}}^O$ is the averaged acceleration vector in AOF; $\bar{\mathbf{a}}^M$ is the averaged measured acceleration vector in ABF.

When the accelerometer is in a stationary position, the vector $\bar{\mathbf{a}}^O$ is equal to the gravitational acceleration vector $\mathbf{g}$ and (2) becomes:

$$\mathbf{g} = \mathbf{T} * \mathbf{K} * \left(\bar{\mathbf{a}}^M - \mathbf{b}\right) \tag{3}$$

Modern digital MEMS accelerometers data are not read in m/s^2, but in units of g. Consequently, the norm of the gravitational acceleration vector is 1 and consequently the equation (3) becomes:

$$1 = \left\| \mathbf{T} * \mathbf{K} * \left(\bar{\mathbf{a}}^M - \mathbf{b}\right) \right\| \tag{4}$$

The averaged measurements in significantly different positions of the accelerometer are recorded. Using (3) and data from these measurements, the fitness function is defined as:

$$f\left(\hat{\boldsymbol{\lambda}}\right)=\sum_{i=1}^{N}\left(1-\left\|\mathbf{T}*\mathbf{K}*\left(\overline{\mathbf{a}}_i^M-\mathbf{b}\right)\right\|\right)^2 \tag{5}$$

where: $\hat{\boldsymbol{\lambda}}=[\hat{\alpha}_{yz},\hat{\alpha}_{zy},\hat{\alpha}_{zx},\hat{S}_x,\hat{S}_y,\hat{S}_z,\hat{b}_x,\hat{b}_y,\hat{b}_z]$ is the vector of the evaluated parameters; $\overline{\mathbf{a}}_i^M$ are the vectors of averaged measurements in different positions; N is the number of positions.

Therefore, a genetic algorithm for minimising of the fitness function (5) is used. The basic parameters of the genetic algorithm are:

- number of individuals in the current generation that survive to the next generation called elite children – 2,
- fraction of the population at the next generation, excluding the elite children from parents of the previous generation - 0.8,
- fraction of the smaller of the two subpopulations of individuals that migrate from one to another subpopulation is 0.2,
- the algorithm stops if the average relative change in the best fitness function value over 50 generations is less than or equal to 10^{-10} or the number of generations reaches 150,
- the population size (number of individuals in each generation) is 200.

The vector of the evaluated parameters is the vector from the last generation for which the fitness function is minimal. The outputs of the genetic algorithm are random variables, so the mean values from applying the algorithm 20 times with the same input data are taken as estimates. The algorithm is applied iteratively 3 times. Before the second and third iterations, new lower and upper bounds of parameters are calculated:

$$\begin{aligned}\boldsymbol{\lambda}_{2\min}&=\hat{\boldsymbol{\lambda}}_1-5\boldsymbol{\sigma}_{\hat{\boldsymbol{\lambda}}_1} & \boldsymbol{\lambda}_{2\max}&=\hat{\boldsymbol{\lambda}}_1+5\boldsymbol{\sigma}_{\hat{\boldsymbol{\lambda}}_1}\\ \boldsymbol{\lambda}_{3\min}&=\hat{\boldsymbol{\lambda}}_2-15\boldsymbol{\sigma}_{\hat{\boldsymbol{\lambda}}_2} & \boldsymbol{\lambda}_{3\max}&=\hat{\boldsymbol{\lambda}}_2+15\boldsymbol{\sigma}_{\hat{\boldsymbol{\lambda}}_2}\end{aligned} \tag{6}$$

where: $\hat{\boldsymbol{\lambda}}_1$ is the mean vector from 20 times application of the genetic algorithm on the first iteration; $\boldsymbol{\sigma}_{\hat{\boldsymbol{\lambda}}_1}$ is the vector of standard deviations of estimated parameters of 20 times application of the genetic algorithm on the first iteration; $\hat{\boldsymbol{\lambda}}_2$ is the mean vector of 20 times application of the genetic algorithm on the second iteration; $\boldsymbol{\sigma}_{\hat{\boldsymbol{\lambda}}_2}$ is the vector of standard deviations of estimated parameters of 20 times application of the genetic algorithm on the second iteration.

The estimated parameters are used in equation (2) to find estimated acceleration.

3. RESULTS AND DISCUSSION

The first step of the simulation is to generate a set of accelerometer positions. The second step is to add errors in order to create a simulation of measurements. The third step is to apply the iterative procedure described above.

The simulation of a set of accelerometer positions is done by Euler rotations about each axis as it is described in [3] and the total number of the positions of the accelerometer is the sum of the number of rotations about each axis(N_x, N_y, N_z). The second and third steps of the simulation are the same as in [8].

One hundred simulations for each number of accelerometer positions are done to examine estimation errors. Different random rotations, biases and scale factors are generated in each simulation. The vector of estimation errors
$\Delta\boldsymbol{\lambda} = [\Delta\alpha_{yz}, \Delta\alpha_{zy}, \Delta\alpha_{zx}, \Delta S_x, \Delta S_y, \Delta S_z, \Delta b_x, \Delta b_y, \Delta b_z]$ is obtained for each simulation:

$$\Delta\boldsymbol{\lambda} = \boldsymbol{\lambda} - \hat{\boldsymbol{\lambda}} \tag{7}$$

where $\boldsymbol{\lambda}$ is the vector of true parameters.

The errors of overall bias are also calculated:

$$\Delta b = \sqrt{\left(b_x - \hat{b}_x\right)^2 + \left(b_y - \hat{b}_y\right)^2 \left(b_z - \hat{b}_z\right)^2} \tag{8}$$

To evaluate the accuracy of estimation, the means and standard deviations (STD) of errors are obtained. The results for non-orthogonality terms when the numbers of rotation about axes are $N_x = 12$, $N_y = 12$ and $N_z = 0$ are shown in Fig. 1. The results show that the biggest means and standard deviations of errors are after the first iteration. There are small differences between standard deviations of errors for second and third iterations. It is obvious that the means and standard deviations of $\Delta\alpha_{yz}$ are much bigger than those of other non-orthogonality terms, because of the absence of rotation about z-axis in different accelerometer positions.

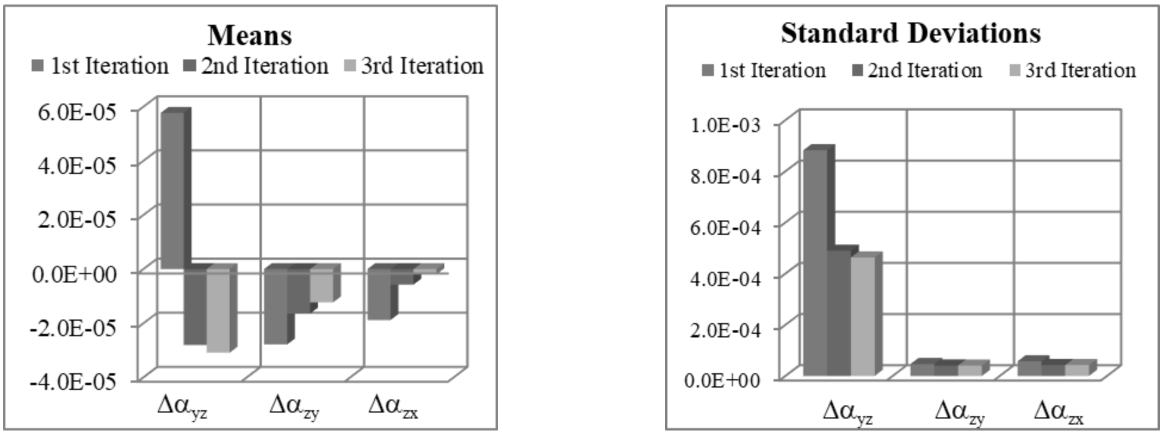


Fig. 1. Means and STD of non-orthogonality terms estimation errors for N_x=12, N_y=12, N_z=0

The results for scale factors when the numbers of rotation about axes are $N_x = 12$, $N_y = 12$ and $N_z = 0$ are shown in Fig. 2.

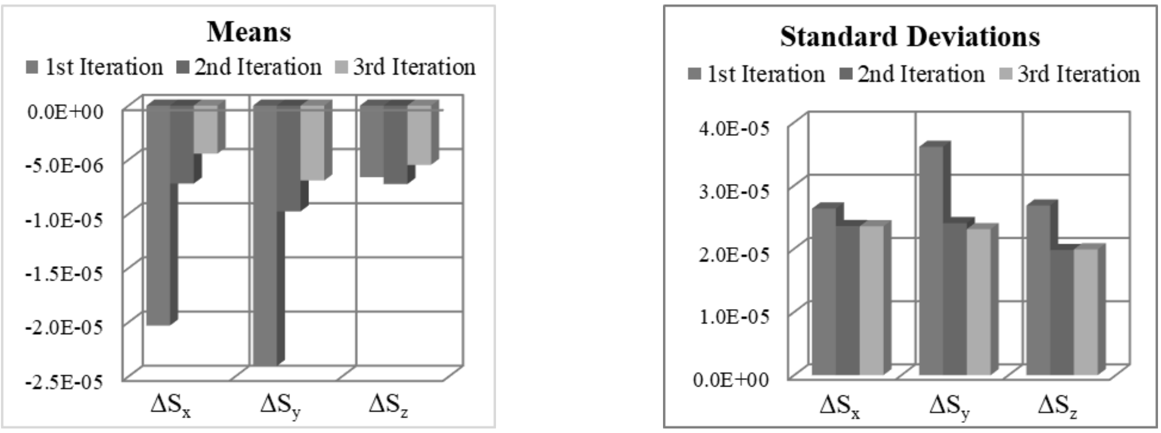


Fig. 2. Means and STD of scale factors estimation errors for N_x=12, N_y=12, N_z=0

Figure 2 shows that the means and standard deviations are smaller after the third iteration, but there is no significant difference between the second and the third iterations. The means of errors are below 10^{-5} for the second and the third iterations. The values of the standard deviations are below 2.5×10^{-5} for the same two iterations.

The results for biases when the numbers of rotation about axes are $N_x = 12$, $N_y = 12$ and $N_z = 0$ are shown in Fig. 3. The results show that the estimation errors are much smaller for second and third iterations. The means of bias estimation errors are less than 10^{-5}g for the second and third iterations while the standard deviations are below 2×10^{-5}g.

The results for overall bias estimation errors when the numbers of rotation about axes are $N_x = 12$, $N_y = 12$ and $N_z = 0$ are shown in Fig. 4. The means of errors are less than 3.5×10^{-5}g for the second and third iterations, and the standard deviations are below 1.6×10^{-5}g.

The results for regarded accelerometer positions show that the estimations of parameters by the proposed genetic algorithm are good, but there are discrepancies in the estimates of non-orthogonality terms.

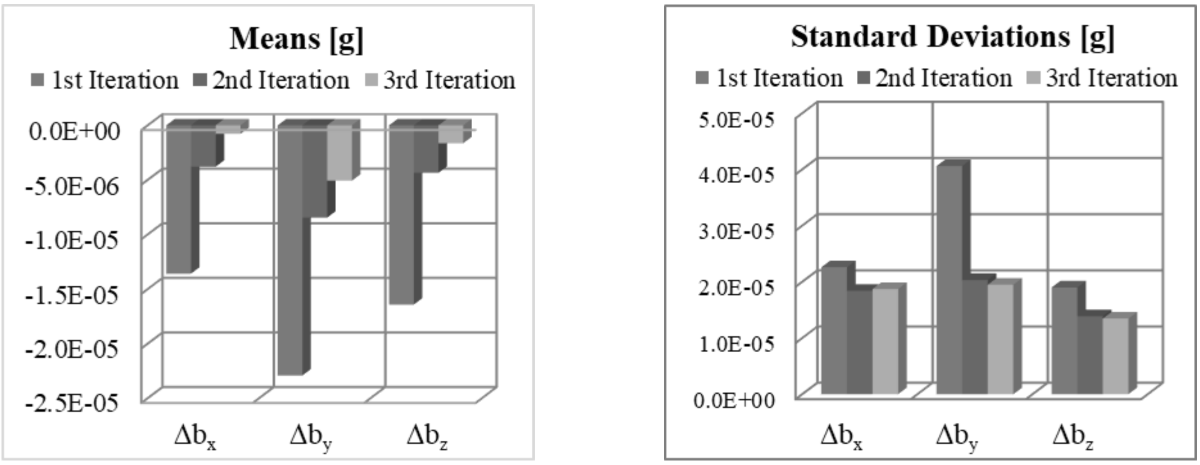


Fig. 3. Means and STD of estimation errors of biases for N_x=12, N_y=12, N_z=0

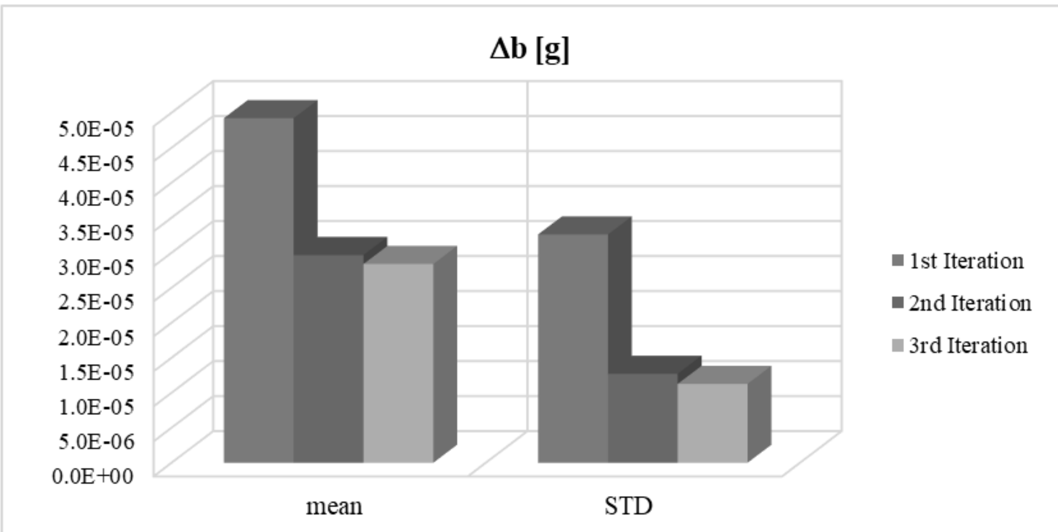


Fig. 4. Means and STD of overall bias estimation errors for N_x=12, N_y=12, N_z=0

The results when the numbers of rotation about axes are N_x=8, N_y=6, and N_z=4 are shown in Fig. 5 to 8.

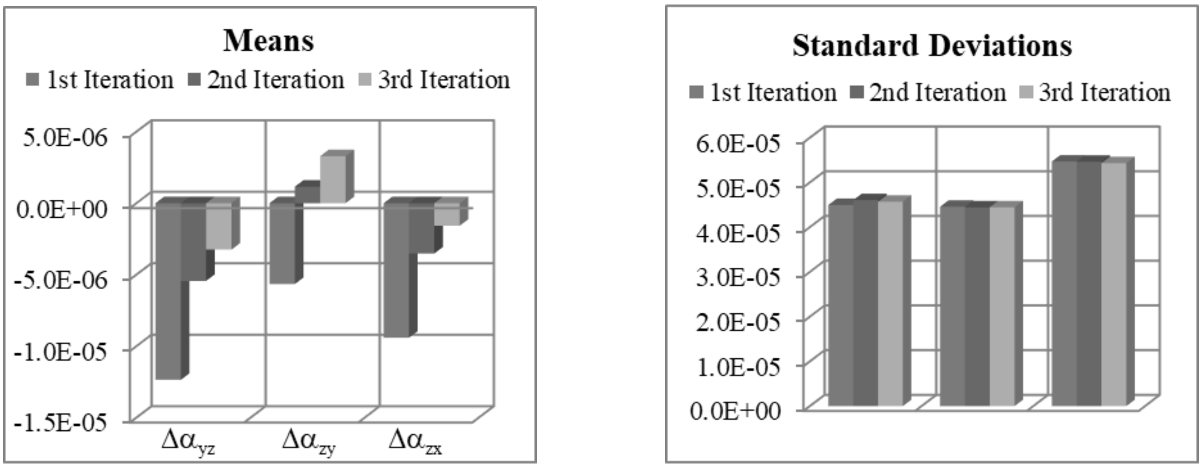


Fig. 5. Means and STD of non-orthogonality terms estimation errors for N_x=8, N_y=6, N_z=4

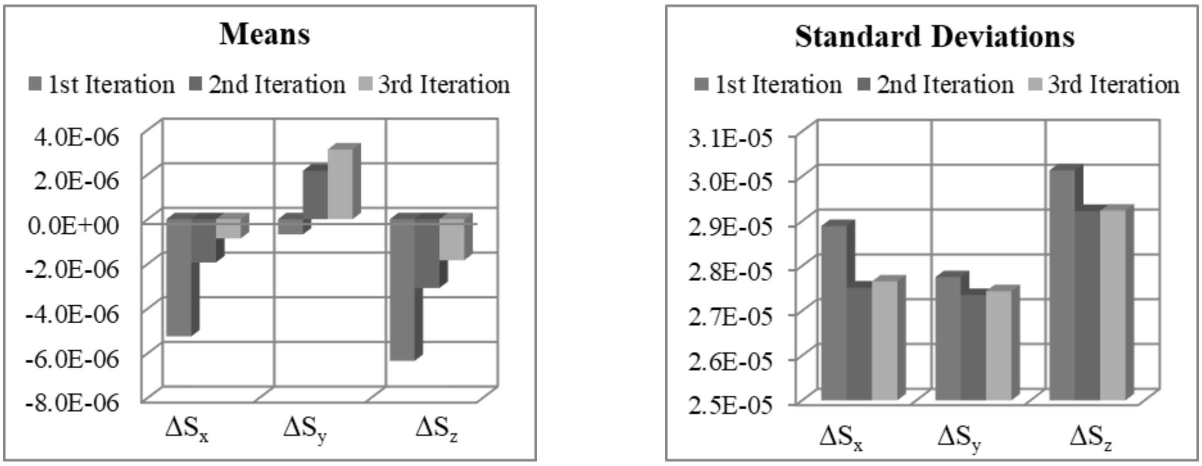


Fig. 6. Means and STD of scale factors estimation errors for N_x=8, N_y=6, N_z=4

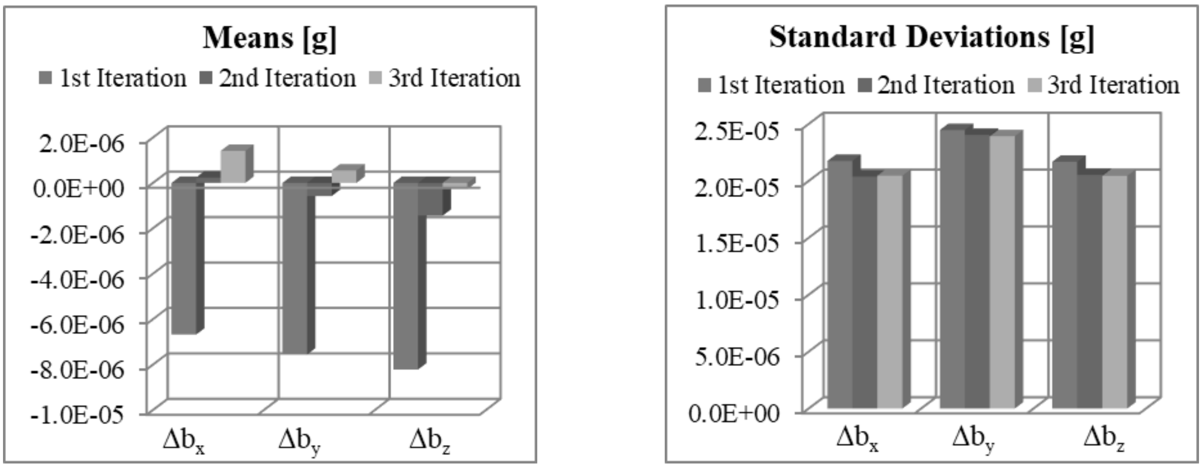


Fig. 7. Means and STD of estimation errors of biases for N_x=8, N_y=6, N_z=4

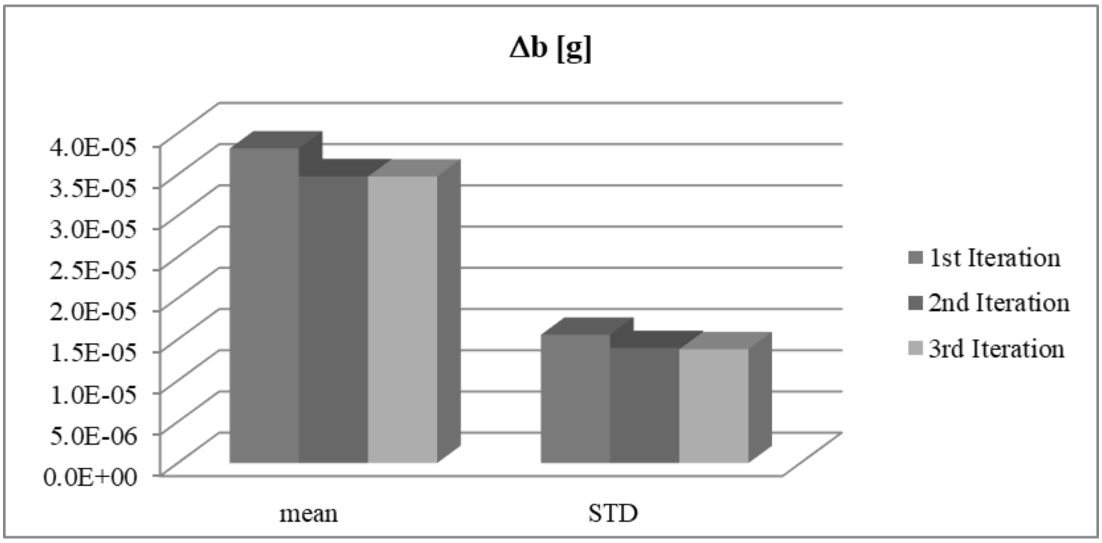


Fig. 8. Means and STD of overall bias estimation errors for N_x=8, N_y=6, N_z=4

Obtained results showed that the accuracy of parameters estimation is slightly higher compared to the previous case. In this case, there are not such big differences between the means and standard deviations of estimation errors of non-orthogonal terms.

The results when the numbers of rotation about axes are N_x=8, N_y=8, and N_z=8 are shown in Fig. 9 to 12.

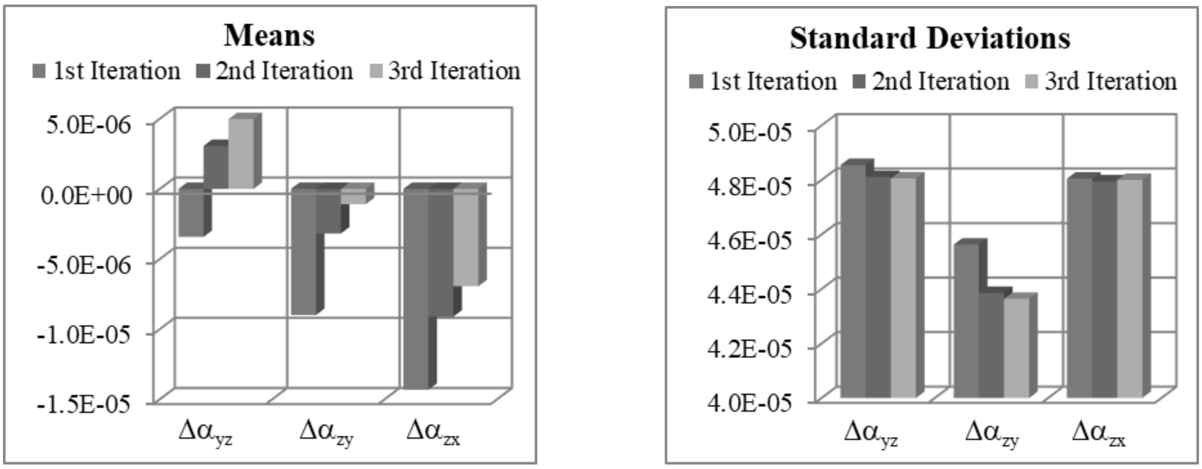
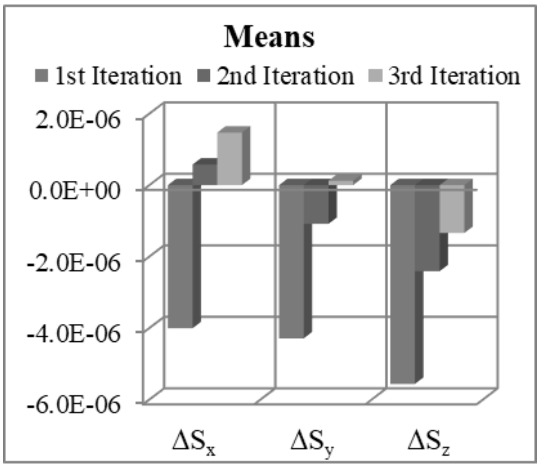


Fig. 9. Means and STD of non-orthogonality terms estimation errors for N_x=8, N_y=8, N_z=8

The obtained results show that the standard deviations of estimation errors are slightly smaller than the previous cases. The improvement is small enough to conclude that there is no need to increase the number of accelerometer positions. The standard deviations of biases estimation errors are practically less than the standard deviation of quantisation error for 14-bit digital MEMS accelerometer with a measurement range of ±2 g. The results prove that there is no need to apply more than two iteration of genetic algorithm.



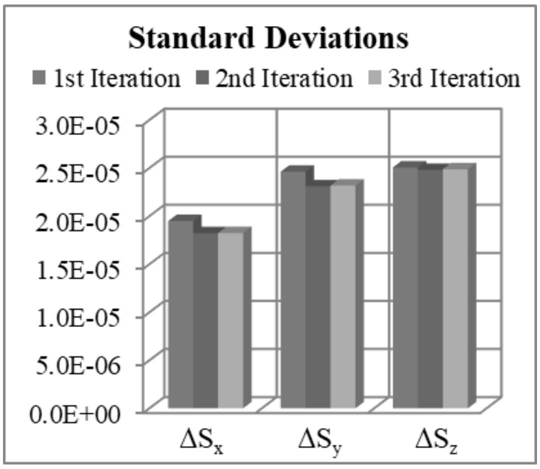
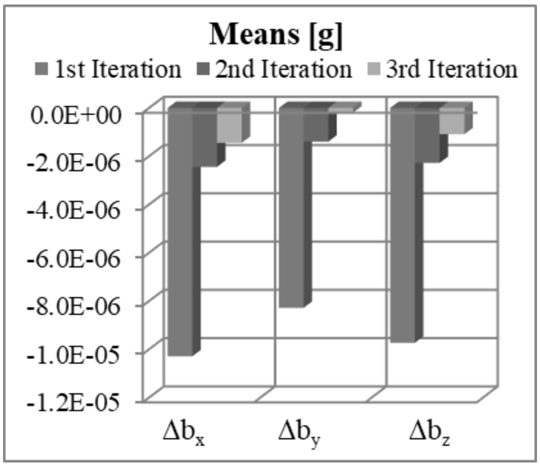


Fig. 10. Means and STD of scale factors estimation errors for N_x=8, N_y=8, N_z=8



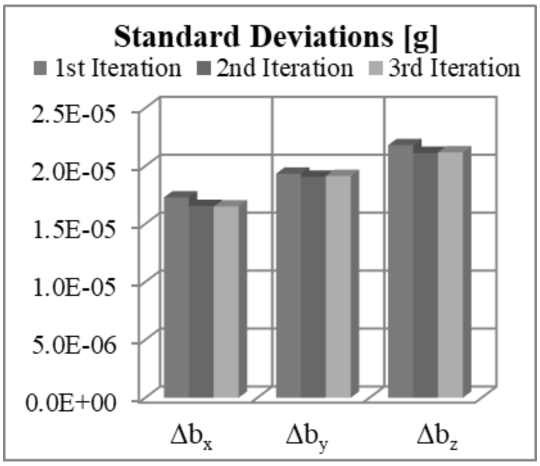


Fig. 11. Means and STD of estimation errors of biases for N_x=8, N_y=8, N_z=8

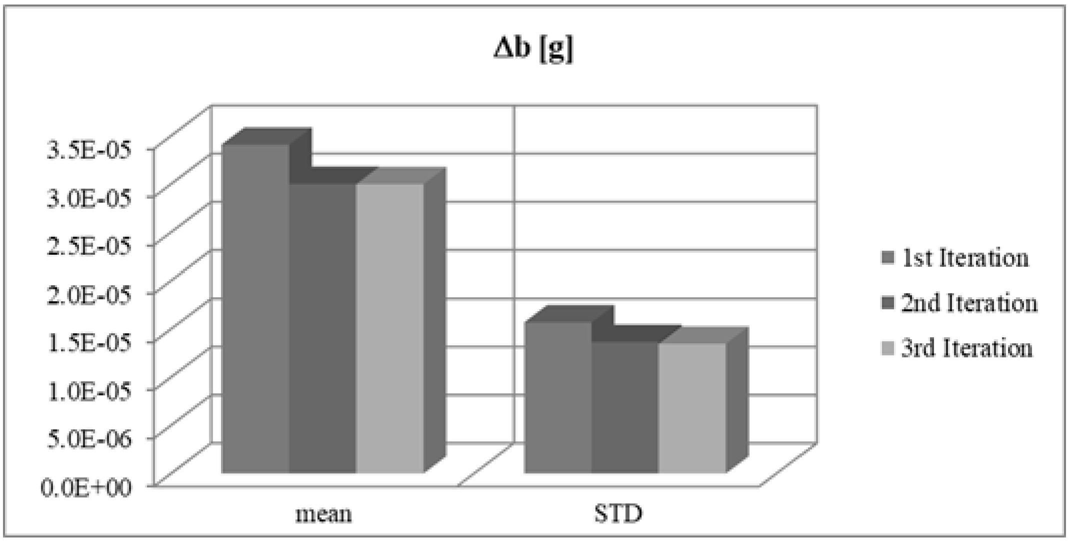


Fig. 12. Means and STD of overall bias estimation errors for N_x=8, N_y=8, N_z=8

The proposed algorithm for static calibration is applied to real 3-axis accelerometer in one MPU 6050 device. The MPU 6050 was put on the head of a camera tripod. The head of the tripod was rotated by hand in 25 different positions. In each position, output data was collected for a period of 20 s. The data rate was 50 Hz and the measurement range was set to ±2 g. The recorded acceleration in each position was averaged in time to obtain a set of 25 measured gravitational acceleration vectors. The measured vectors in accelerometer body frame are shown in Fig. 13.

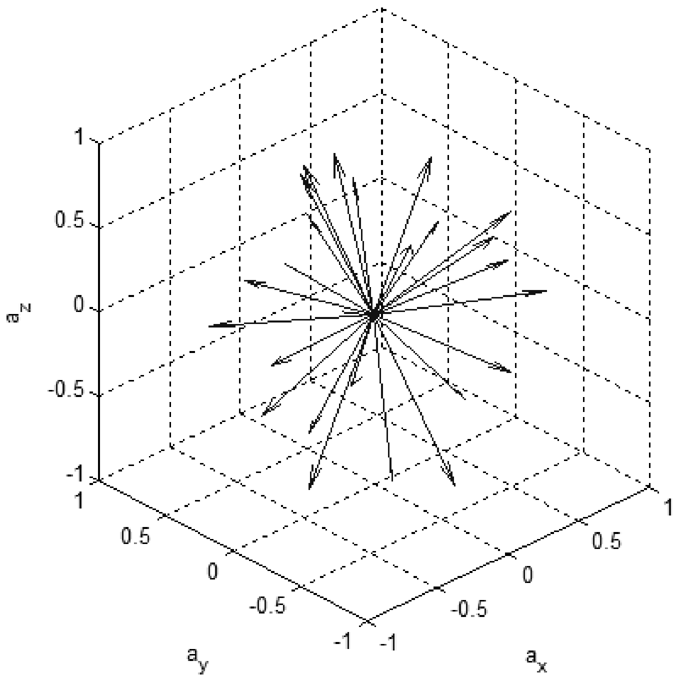


Fig. 13. Measured gravitational acceleration vectors in ABF

The proposed algorithm is used to evaluate the parameters of accelerometer parameters. The values of estimated parameters are shown in Table 1.

Table. 1

Estimated parameters by the proposed algorithm

α_{yz}	α_{zy}	α_{zx}	S_x	S_y	S_z	b_x [mg]	b_y [mg]	b_z [mg]
3.58×10^{-4}	-7.29×10^{-3}	4.62×10^{-4}	1.0006	1.0030	0.9834	27.6253	-5.7079	-78.2870

In the datasheet of MPU 6050, the following values for the initial tolerance of biases are given: b_x=b_y=±50g and b_z=±80g. As seen in Table 1, the obtained estimated values are within these ranges. The differences between estimated biases on each axis are significant.

The same set of data is used to obtain six calibration parameters by the algorithm proposed in [8]. The estimated parameters are shown in Table 2.

Table 2

Estimated parameters by the algorithm proposed in [8]

S_x	S_y	S_z	b_x [mg]	b_y [mg]	b_z [mg]
1.0008	1.0032	0.9835	28.1414	-5.4880	-77.2980

As could be seen from the above tables, the estimated values of biases and scale factors by the two algorithms are slightly different. To evaluate the accuracy of calibration, the following calculations are made.

1. The calibrated measured accelerations on each axis are calculated for all positions using the parameters from the tables.
2. Double digital integration is applied to calibrate accelerations in order to obtain the equivalent path on each axis at each time moment [$s_{ix}(t)$, $s_{iy}(t)$ and $s_{iz}(t)$].
3. The errors in the overall equivalent path for each position are calculated using the following formula $\Delta s_i(t) = \sqrt{s_{ix}^2(t) + s_{iy}^2(t) + s_{iz}^2(t)} - 0.5gt^2$.
4. Averaging over each position is done at each time moment and the mean and standard deviation (σ_S) of errors in are obtained.

The calculated standard deviation of errors of the overall equivalent path without calibration as a function of time is shown in Fig. 14a. While the calculated standard deviation of errors of the overall equivalent path with calibration as a function of time is shown in Fig. 14b.

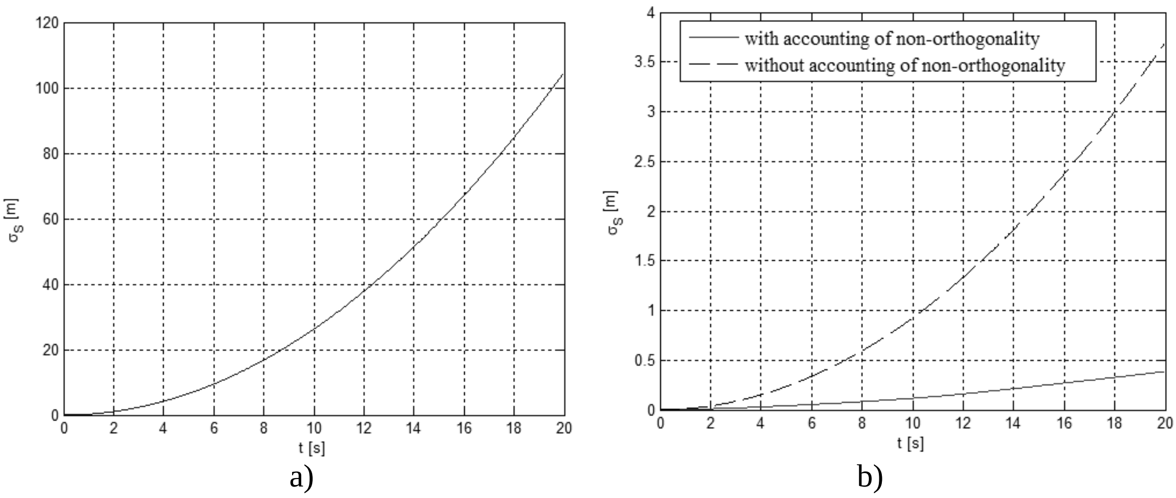


Fig. 14. Standard deviation of errors of the overall equivalent path.

The results show that the standard deviation of errors of equivalent path without calibration at the end of the period of 20 s is over 100 m. The standard deviation of errors in the case of calibration with six parameters is less than 4 m and in the case of calibration with nine parameters is less than 0.4 m. In this way, the accuracy of path calculation, using the proposed algorithm of calibration, is more than 250 times better in comparison to the calculation without any calibration. The improvement in accuracy of the proposed algorithm in comparison to the algorithm with six parameters is about 10 times.

4. CONCLUSIONS

A procedure for MEMS accelerometer static calibration, using a genetic algorithm, is proposed in this paper. A fitness function for the genetic algorithm was defined. The convergence of the estimates to the true values of parameters is very good.

Results from the simulations and real accelerometer calibration showed that the proposed procedure provides very good estimations of accelerometer parameters. The detailed studies showed that two iterations are enough and that it is unnecessary to use more than 25 different positions of the accelerometer. Results of real accelerometer calibration indicated much better accuracy when nine instead six parameters were used.

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