EFFECT OF TORSIONAL VIBRATION ON WOODCHIP SIZE DISTRIBUTION

Summary. Nowadays, there is increasing demand for the use of renewable energy sources such as woodchips. One of the important qualitative parameters of woodchips is the size distribution. The aim of this article is to determine the effect of a woodchipper disc’s torsional vibration on the evenness of woodchip length, as well as propose a mathematical solution to this problem by assuming one harmonic component of disc speed and the uniform feed of chipped material. The presented mathematical solution can be used to determine the unevenness of woodchip length when the parameters of torsional vibration are known.

Keywords: mathematical model; size distribution; torsional vibration; uneven chip length; woodchips

1. INTRODUCTION

Nowadays, there is increasing demand for the use of renewable energy sources such as woodchips. One of the important qualitative parameters of woodchips is size distribution. From the usability point of view, it is important to achieve homogeneous properties of woodchips because heterogeneous material can cause problems with bridging over openings [6], high emissions from burning [9,10], storing and drying [7].

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Woodchip size distribution is affected by dimensional inaccuracies in the woodchipper, feeding speed, geometry of the cutting tool and the properties of the chipped material [1,2].

We assume that woodchip size distribution may be also affected by torsional vibration of the woodchipper’s disc, especially in the case of the inappropriate tuning of the mechanical system drive. The purpose of this paper is to mathematically express the influence of the torsional vibration of the disc chipper on the unevenness of woodchip length.

2. KINEMATICS OF THE WOODCHIPPER DISC

The use of a disc chipper, whose principle is illustrated in Fig. 1, is considered. The disc (1) rotates with the mean angular speed \( \omega \) [rad/s\(^{-1}\)]. The disc has \( z \) knives (2) applied uniformly to the face of the disc. All consecutive knives are rotated relative to one another by the pitch angle \( \varphi_N \). Material with a constant width (3) is fed to the knives at the uniform feeding speed \( v_f \) [mm·s\(^{-1}\)], the value of which must be adjusted so that the chip is cut before touching the disc face in order to avoid energy loss from friction [1].

![Fig. 1. Scheme of the disc chipper](image)

The length of chip \( h_s \) [mm] depends on the disc angular speed \( \omega \), the feeding speed \( v_f \) [mm·s\(^{-1}\)] and the number of knives \( z \). The disc angle is labelled as \( \varphi \) [rad]. The time course of the disc angle is shown in Fig. 2. The disc rotates at the mean angular speed \( \omega \) and simultaneously performs a harmonic torsional vibration with the angular frequency \( i\cdot\omega \). The nominal cutting frequency with the constant angular speed \( \omega \) is equal to \( z\cdot\omega \).

The disc angle, according to Fig. 1, can generally be expressed by the following equation:

\[
\varphi = \omega \cdot t - \varphi_0 \cdot \sin(i \cdot \omega \cdot t - \gamma)
\]  

(1)

where:
- \( t \) - time [s]
- \( \varphi_0 \) - disc angle amplitude [rad]
- \( i \) - order of torsional vibration [-]
- \( \gamma \) - phase of torsional vibration [rad]
The angular velocity $\dot{\varphi}$ [rad·s$^{-1}$] is obtained as a time derivation of the disc angle (1):

$$\dot{\varphi} = \omega - \varphi_0 \cdot i \cdot \omega \cdot \cos(i \cdot \omega \cdot t - \gamma)$$

(2)

For the limit disc angle amplitude $\varphi_0 = \Phi_0$, the stopping of the disc is considered. For larger amplitudes, the angular velocity of the disc can reach negative values. By fitting zero angular velocity for $t=0$ and $\gamma=0$ into Eq. (2), we obtain the value of limit disc angle amplitude:

$$\Phi_0 = \frac{1}{i}$$

(3)

The relative size of torsional vibration $k$ [-] can be expressed with the ratio of the disc angle amplitude $\varphi_0$ to the limit disc angle amplitude $\Phi_0$ as:

$$k = \frac{\varphi_0}{\Phi_0}$$

(4)

where $k \in (0; 1)$.

According to Fig. 2, pitch angle $\varphi_N$ [rad] can be computed as:

$$\varphi_N = \frac{2\pi}{z}$$

(5)
In turn, the disc’s angle of rotation during one period of torsional vibration can be computed as:

$$\varphi_K = \frac{2\pi}{i} \tag{6}$$

Then, the nominal period of cutting $T_N$ [s] can be computed from the nominal cutting frequency:

$$T_N = \frac{2\pi}{z \cdot \omega} \tag{7}$$

In turn, the period of torsional vibration $T_K$ [s] can be computed from the frequency of torsional vibration:

$$T_K = \frac{2\pi}{i \cdot \omega} \tag{8}$$

The woodchip length $h_S$ [mm] can be determined from the feeding speed $v_f$ and time between the moments of touching the chipped material by two consecutive knives $T_S$ [s] corresponding to the pitch angle $\varphi_N$, as shown in Fig. 1 and Fig. 2, by expression:

$$h_S = T_S \cdot v_f \tag{9}$$

The nominal woodchip length $h_N$ [mm] can be computed as:

$$h_N = T_N \cdot v_f \tag{10}$$

As the times $T_S$ are not constant and differ from nominal cutting period $T_N$ (see Fig. 2), the woodchip length will also not be constant. Therefore, we can say that torsional vibration may cause woodchip length unevenness.

3. COMPUTING WOODCHIP LENGTH UNEVENNESS

Woodchip length unevenness can be expressed by the proportional chip length $\nu [-]$, as a ratio of the woodchip length $h_s$ (9 to the nominal woodchip length $h_N$ (10):

$$\nu = \frac{T_S}{T_N} \tag{11}$$

The relationship in (11) shows that the proportional chip length $\nu$ depends on times $T_S$ and $T_N$. 
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Next, a parameter of the frequency ratio \( \eta \) [-], representing the ratio of the frequency of torsional vibration to the cutting frequency, is introduced:

\[
\eta = \frac{i}{z} \tag{12}
\]

To simplify the derivation of relationships, we introduce the dimensionless parameters of torsional vibration and woodchip length. The dimensionless representation of torsional vibration is shown in Fig. 3, where dimensionless time \( s \) is related to the period of torsional vibration \( T_K \):

\[
s = \frac{t}{T_K} \tag{13}
\]

and a dimensionless rotation angle \( u \) is related to the disc’s angle of rotation during one period of torsional vibration \( \varphi_K \):

\[
u = \frac{\varphi}{\varphi_K} \tag{14}
\]

Dimensionless woodchip length \( S \) will be related to the nominal period of torsional vibration \( T_K \):

\[
S = \frac{T_S}{T_K} \tag{15}
\]

The nominal dimensionless woodchip length can be obtained as a ratio of nominal period of cutting \( T_N \) (7) to the period of torsional vibration \( T_K \) (8), which will be equal to frequency ratio \( \eta \), see relationship (12):

\[
S_N = \eta \tag{16}
\]

The dimensionless amplitude of torsional vibration \( u_0 \) can be obtained as a ratio of torsional vibration amplitude \( \varphi_0 \), expressed in the formula in (4), to the disc’s angle of rotation during one period of torsional vibration \( \varphi_K \) (6):

\[
u_0 = \frac{k}{2\pi} \tag{17}
\]

Then, the proportional woodchip length can be computed from dimensionless values \( S \) and \( S_N \) as:

\[
\nu = \frac{S}{S_N} \tag{18}
\]
The woodchip length will reach the limit values when it is located symmetrically around points where the actual speed of the disc is at its maximum (Fig. 3a) or minimum (Fig. 3b).

![Fig. 3. Dimensionless representation of torsional vibration](image)

The actual value of the length of each woodchip will lie between these limit values. The range $(0; 1)$ in Fig. 3, on the horizontal axis, corresponds to one period of torsional vibration $T_K$, while, on the vertical axis, it corresponds to the disc’s angle of rotation during one period of torsional vibration $\varphi_K$.

According to Fig. 3, we can express the nominal dimensionless woodchip length as:

$$S_N = S_{1,2} \pm 2 \cdot u_0 \cdot \sin(\pi \cdot S_{1,2})$$  \hspace{1cm} (19)

and the envelope of limit values of dimensionless woodchip length as:

$$S_{0,1,2} = S_N \mp 2 \cdot u_0$$  \hspace{1cm} (20)

By modifying Eq. (19), we obtain a formula for the limit values of dimensionless woodchip length:

$$S_{1,2} = S_N \mp 2 \cdot u_0 \cdot \sin(\pi \cdot S_{1,2})$$  \hspace{1cm} (21)

Now, we can express the limit values of the proportional woodchip length as the ratio of $S_{1,2}$ from Eq. (20) to $S_N$ from Eq. (16), and by using the formula in (17 as $u_0$):

$$\nu_{1,2} = \frac{k}{\eta \cdot \pi} \cdot \sin(\pi \cdot S_{1,2})$$  \hspace{1cm} (22)

By introducing substitutions into the formula in (22, the limit values of the proportional woodchip length $\nu_{1,2}$ are finally obtained as:
where:

\[ \delta_0 \] - the limit envelope value of the proportional woodchip length:

\[ \delta_0 = \frac{k}{\pi \cdot \eta} \]  \hspace{1cm} (24)

\[ \xi_{l,2} \] - the limit envelope value of proportional woodchip length:

\[ \xi_{l,2} = \frac{1}{\eta} \sin(\pi \cdot S_{l,2}) \]  \hspace{1cm} (25)

However, we need to express the value of \( \xi_{l,2} \) depending on \( S_N \). Therefore, the inverse function to (19) should be fit into Eq. (25. The graphical solution is shown in Fig. 4. The functions \( \xi_{l,2} \) are periodic, with the period of frequency ratio \( \eta = 2 \). As the shape of \( \xi_1 \) and \( \xi_2 \) functions are identical, and the phase shift between them is equal to 1, only a half-period is shown in Fig. 4. We also do not need to know which line exactly represents \( \xi_1 \) and \( \xi_2 \), because we only need to know that the actual proportional woodchip length will lie between these limit values.
Fig. 4. Limit envelope value of the proportional woodchip length

Fig. 5 shows the limit values of the proportional woodchip length, according to the formula in (23, depending on frequency ratio $\eta$, in terms of different relative sizes of torsional vibration $k$. 

\[(j + \eta) [\cdot], j \in \{0,1,2;\ldots\}\]
4. DISCUSSION

According to Fig. 5, the influence of the following torsional vibration parameters on woodchip length unevenness can be stated thus:

(1) Frequency ratio $\eta$

If the frequency ratio between the cutting frequency and the torsional vibration frequency is an integer, unevenness in woodchip length cannot arise.

This can be accomplished by using a mesh drive with a proper gear ratio (e.g., gear, timing belt, chain drives). With a friction drive, due to slippage, this goal cannot be accomplished.

The frequency ratio is influenced by the following parameters:

(1a) Frequency of torsional vibration excited by the woodchipper’s disc

The frequencies of the torsional vibration harmonic components excited by the disc will be an integer multiple of the number of knives. This means that the unevenness of woodchip length, due to the woodchipper’s disc excitation, cannot theoretically arise.

(1b) Frequency of torsional vibration excited by the drive

This includes harmonic components excited by the engine (most likely, a piston combustion engine) or fluctuations in the gear ratio (e.g., due to the use of a cardan joint, shaft misalignment) [4]. For these influences, it is advisable to keep the engine torque constant (use of electric motor) or to maximize the order of the main harmonic component (choosing a piston engine with a larger number of cylinders).

(1c) Frequency of self-excited vibration

The frequency of self-excited vibration is close to the natural frequency of the mechanical system [3]. In this respect, it is advantageous to have the frequency of self-excited torsional
vibration (natural frequency) as high as possible. As such, it is necessary to avoid an integer ratio of natural frequency to cutting frequency in order to avoid resonance in the system [11].

(1d) Number of knives

Woodchip length unevenness reaches the highest values in the case of low values of the frequency ratio, i.e., for low frequencies of torsional vibration and a high number of knives.

Therefore, it is advantageous to choose as low a number of knives as possible.

(2) Size of torsional vibration amplitude

Woodchip length unevenness increases depend on the rising torsional vibration amplitude. Therefore, it is the best to keep the amplitude as low as possible. The highest value of vibration amplitude occurs in the case of resonance, i.e., when the frequency of the exciting torque is equal to the mechanical system’s natural frequency. This can be avoided by the proper tuning of the system’s dynamic parameters (i.e., torsional stiffness, mass moment of inertia, damping coefficient) [5].

4. CONCLUSION

The method presented in this paper is suitable for determining woodchip length unevenness caused by torsional vibration of the disc chipper. In order to use this method, it is necessary to know the parameters of torsional vibration (frequency and amplitude), which can be obtained theoretically by dynamic analysis or by measurement.

In the future, it will be necessary to examine the realistically achievable range of woodchip length unevenness caused by torsional vibration, based on the dynamic analysis of currently produced disc chippers.

References

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