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ANALYSIS OF SELECTED SOLUTIONS AND METHODS TO LIMIT THE UNEVEN LOAD DISTRIBUTION ON THE TOOTH WIDTH

Summary. This article is dedicated to the subject of gearboxes, with a specific focus on the load distribution on the tooth. The aim of the study was to review the selected solutions and methods that can contribute to aligning the load on the width of the tooth. The structure and operation of each solution are described in detail. The paper presents the possible benefits of using the described solutions.
Keywords: gear wheel; contact stress; load distribution

1. INTRODUCTION

Due to the numerous advantages involved, gear transmissions are used in most current drive systems. Gears can be used in a wide range of applications, such as land vehicles, marine propulsion systems, aviation engines or machinery drives in technological processes. Regardless of purpose, the gearboxes should be characterized by high reliability and durability. A very important factor influencing the previously mentioned characteristics of transmission is load distribution over the tooth width, which occurs during the transmission operation. Any unevenness of the load along the teeth contact line contributes to a local

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increase in transmission load, which results in the deterioration of its durability and carrying capacity. The article reviews the innovative construction solutions aimed at counteracting the effects of an uneven load on the tooth width [1, 2, 3, 5].

2. LOAD DISTRIBUTION ON THE WIDTH OF THE TOOTH AND ITS IRREGULARITY

Under real operating conditions, the load distribution along the teeth contact line is not even, while the accompanying contact stresses are the main factors that determine the durability of transmission. In order to equalize the load distribution on the width of the meshing, a modification of the tooth trace direction can be used.

Unfortunately, there are also numerous difficult-to-avoid factors that influence the load distribution along the teeth contact line, resulting in increased contact stress and the uneven distribution of load on the tooth width [1, 3, 4, 5, 8]. According to [3, 5], the factors influencing the load distribution on the tooth's width can be classified as follows:

- geometry of meshing
- meshing deviations
- deviations of the buildings caused by deviations of the axle position
- internal bearing clearance
- the process of lapping the gear unit
- misalignment and parallelism of transmission shafts
- stiffness of the wheels (teeth and hub), shafts, bearings, transmission and its foundation
- compensating structural means
- additional shaft load (e.g., pulley)

As a measure of the unevenness of the load on the width of the meshing, the coefficient of unevenness of the load distribution *K* can be defined. Factor *K* is determined as the ratio of the local maximum tooth load per unit length to the mean value of load, which is calculated assuming an even load distribution. The value of the coefficient *K* can be calculated from the following relation, as reported in [1]:

$$K = \frac{p_{max}}{p_m} = \frac{f_{max}}{f_m} \tag{1}$$

where:

p_{max} – maximum load per unit of wheel width

p_m – average load per unit of wheel width

f_{max} – maximal local deformation of the tooth

f_m – average tooth deformation on wheel width

The main determinant of the distribution of the load on the tooth width is the resulting deviation of the teeth contact line in the plane of action $F_{\beta y}$. $F_{\beta y}$ deviation is defined as the distance of the sides of the teeth that cooperate in the frontal direction. Its measurement is made in the situation of entering into meshing of cooperating pairs of teeth. The value of deviation $F_{\beta y}$ can be determined on the basis of the relation shown in [3]:

$$F_{\beta_y} = F_{\beta_x} - y_\beta \tag{2}$$

where:
$F_{\beta x}$ – initial contact line deviation without taking lapping into account
y_{β} – a lapping value that decreases the initial deviation of the contact line as a result of lapping teeth during the operation

3. REVIEW AND ANALYSIS OF SELECTED STRUCTURAL SOLUTIONS TO REDUCE THE IRREGULARITY OF LOAD DISTRIBUTION ON TOOTH WIDTH

Appropriate structural solutions may be used to reduce undesirable effects of uneven load distribution on the width of the tooth. The following section presents and describes innovative solutions for reducing the adverse effects of the uneven distribution of load.

3.1. Divided satellite gear wheels of planetary transmission

In planetary gearboxes, the satellite wheels work with the sun gear and internally toothed wheel. Often the sun wheel and satellites have a relative wide width, due to considerable surface pressures between the cooperating teeth. For fixing satellites on the journal axes, two bearings (ball or roller) are most commonly used. Such a way of settling the wheels and their considerable wide width, despite the high degree of performance and assembly of the transmission, contribute to the uneven load distribution on the width of the tooth. In order to reduce this unevenness, an innovative design of satellite wheels was proposed in [9]. These solutions consist of dividing one wide satellite wheel in a plane perpendicular to its axis of rotation into a few narrower separate wheels. The total width of the newly formed wheels is the same as the width of a single wide wheel. In addition, one oscillating bearing on each of the narrower wheels is used. Such bearing independence and the additional possibility of tilting the wheel significantly increase its give and allow a more favourable load distribution on the tooth width [7, 9].

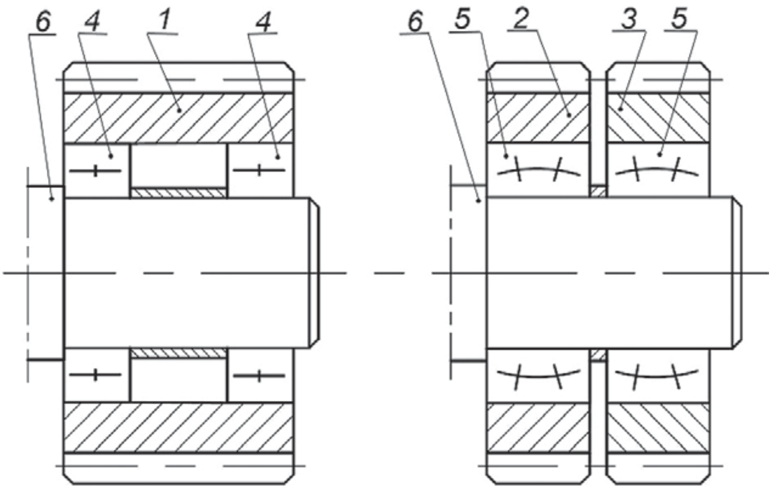

Fig. 1. Diagram of a portion of a satellite wheel with bearing. Description of indications: 1) toothed ring of the classic wheel; 2-3) toothed rings of narrower wheels formed; 4) bearings; 5) self-adjustable bearings; 6) journal of the satellite wheels [7]

3.2. Oscillating wheel setting on the shaft using four cylinders

A solution to limit uneven load distribution on tooth widths can be found in the patent (PL 379605). This patent contains an application of a constructional solution consisting of a pivotally connected shaft journal with a hub. Four cylinders are arranged perpendicular to the axis of rotation of the shaft. They are placed in the holes on the surface of the journal at 90° intervals. The other end of each cylinder is located in the groove on the inner surface of the hub. The hub grooves have a rectangular cross section and are parallel to the axis of rotation. The surface of the cylinder face, which is in the hub groove, is rounded by a radius. The length of the radius corresponds to the distance from the bottom of the groove to the axis of rotation of the journal. The proposed construction solution is designed to allow the wheel hub to tilt at an angle to the shaft journal on which it is mounted. This will increase wheel susceptibility, which in turn will improve the load distribution on its width [10].

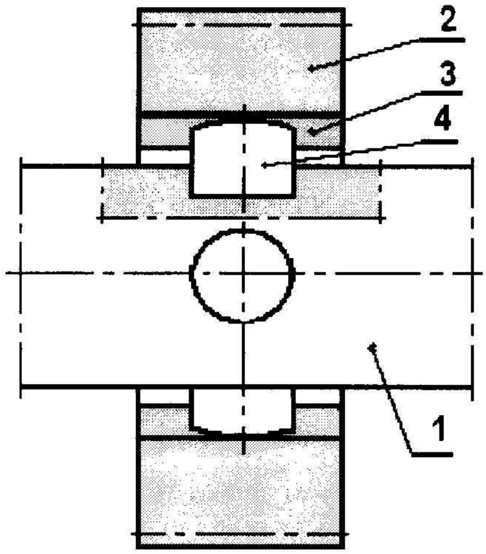

Fig. 2. Diagram of oscillating wheel setting on the shaft, using four cylinders. Description of indications: 1) shaft's journal; 2) toothed gear; 3) groove with rectangular cross section; 4) cylinder [10]

For the above solution, a three-dimensional model has been created. It shows the features of the analysed solution and simulates its operation in different hub positions. For further consideration, an angle φ was defined between the axes of the cross (obtained from the four cylinders mentioned above) and their projection on the plane containing the shaft axis and the gear axis. This plane is formed at a non-zero angle between the shaft and gear axles.

It has been observed that, in the case of non-zero angles between the axis of the shaft journal and the axis of the hub and the non-zero values of the angle φ, there is overlapping of the side faces of the rollers and the inner surface of the hub grooves. At a constant angle between the shaft axis and the axis of the hub, this phenomenon reaches a maximum value of φ = 45°. This phenomenon increases with the wheel hub angle and is illustrated in Fig. 3. The presented situation can adversely affect the durability of the joint and significantly accelerate the degradation of the roll surfaces and hub grooves.

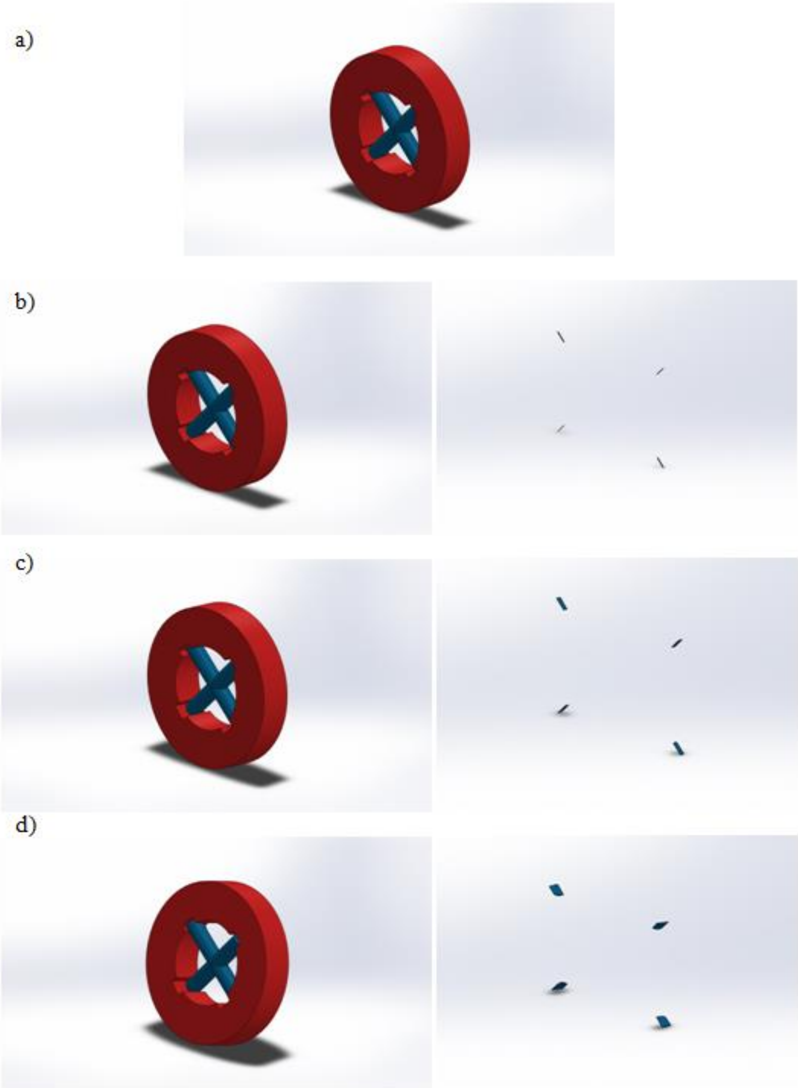

Fig. 3. Simulation of the hub oscillating operation: a) three-dimensional model of the joint showing the main characteristics of the analysed solution; b) the angle between the journal axis and the hub axis is 1°; c) the angle between the journal axis and the hub axis is 5°; d) the angle between the journal axis and the hub axis is 15°

Figure 4 shows the overlapping volume of the side rolls and the inner surface of the hub grooves Vk as a function of the angle between the shaft rotation axis and the axis of the gear. In the illustrated case, the diameter of the four-cross cylinder was 10 mm, the length of the two cylinders was 60 mm and the diameter of the hole in hub was 52 mm.

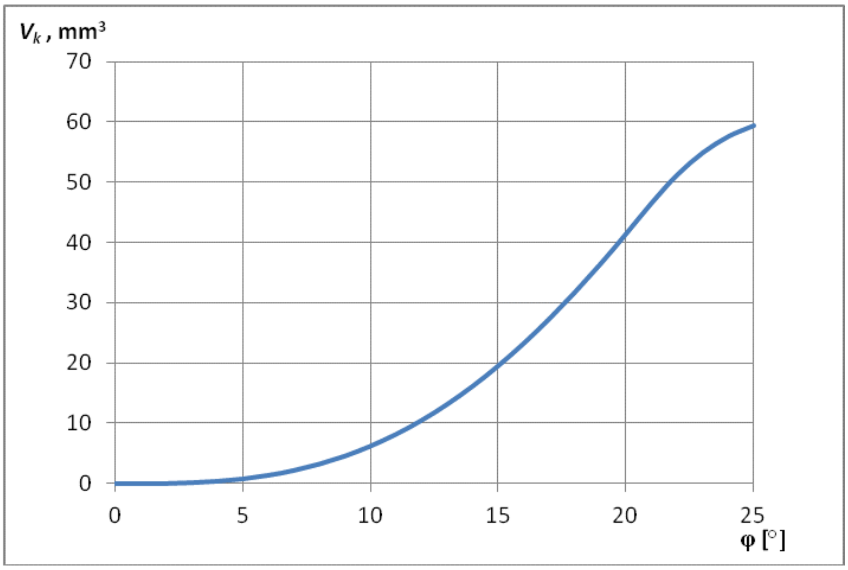

Fig. 4. Overlapping volume Vk diagram as a function of angle φ between the journal axis and the hub axis

The dimensionless coefficient of the overlapping volume Vk (Fig. 5) and the volume of the crosspiece fragments, that is, the hub grooves Vf, were also proposed. The Vk/Vf coefficients as a function of the angle between the axis of the shaft journal and the axis of the hub φ are shown in Fig. 6.

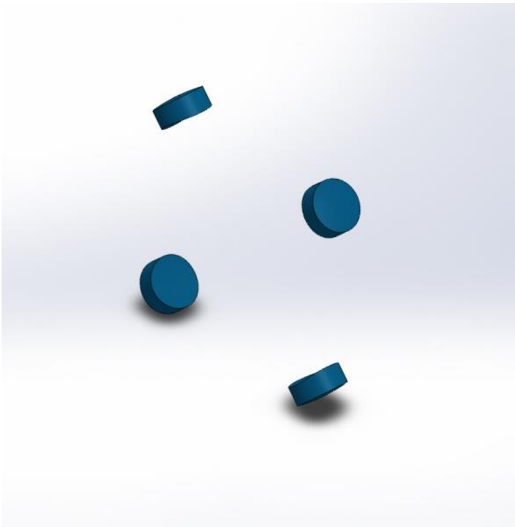

Fig. 5. Fragments of crosspieces in the hub grooves described as V_f

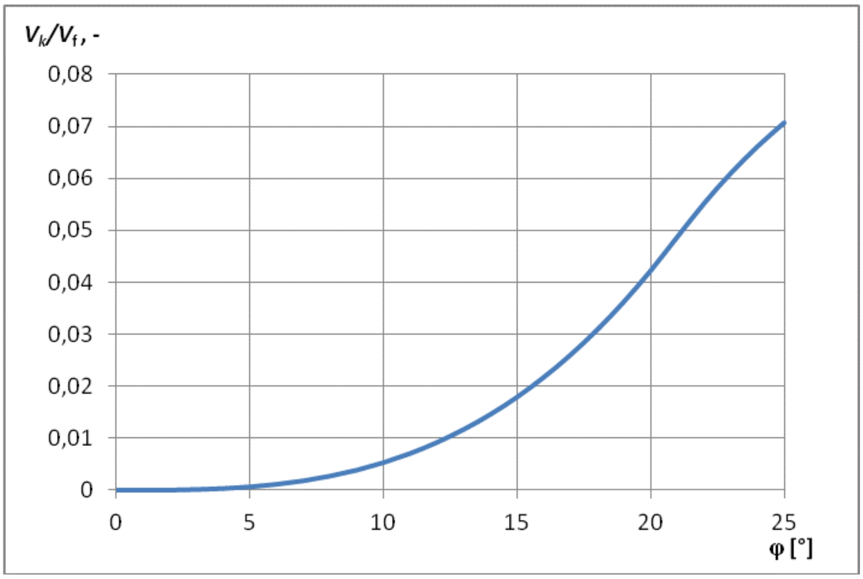

Fig. 6. Graph of Vk/Vf ratio as a function of the angle of hub deflection φ

3.3. Eccentric setting of the shaft's bearings

One method of limiting the uneven load distribution on the tooth width, without interfering with wheel construction, is the eccentric mounting of the shaft's transmission bearings. Thanks to eccentric bearings, it is possible to regulate the non-parallelism and misalignment of the transmission axis, which also significantly affects the load distribution on the width of the wheels mounted on the adjustable shafts. In addition, this adjustability reduces the adverse impact of the elastic deformations of the load-carrying elements and the deviations. The eccentric mounting of the shaft bearings presented in [5] is particularly beneficial in the case of high-power transmission with significant geometric dimensions. It is also worth emphasizing that, compared to other solutions, this method does not significantly increase the cost of transmission, but improves its strength properties [5, 6].

3.4. Wheel construction with malleable elements

Patents US 2307129 [11] and US 307705 [12] illustrate the construction of a gear, based on the separation of a cylindrical hub from a toothed ring by use of an additional ring made of a malleable material. The radius of the cylinder base was not accurately determined by the authors. The ring can be one element or, as in the case of patent US 307705, two separate and respectively narrower elements. The frictional force occurring on the contact surface of the ring with the hub and the ratchet enables the transfer of torque. In addition, in patent US 2307129, the authors applied two discs attached to both side surfaces of the wheel by a screw connection. Discs are designed to limit the excessive ring and wheel rim movements along the shaft rotation axis. The use of a soft material in the toothed wheel structure allows for a slight angular deviation in the rim, relative to the hub, which translates into increased wheel give and can contribute to a more favourable load distribution on its width.

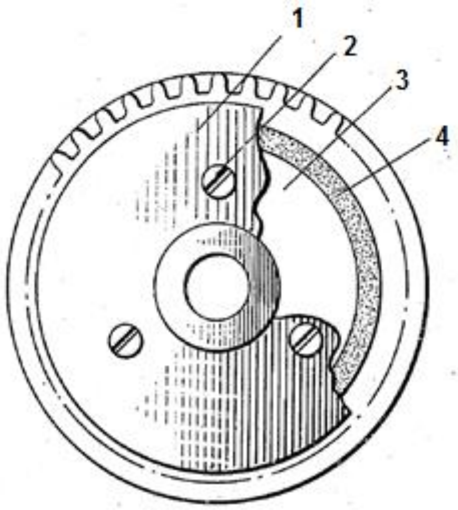

Fig. 7. Scheme of construction of wheel with a malleable ring. Description of indications: 1) side discs; 2) screw connection; 3) wheel hub; 4) susceptible ring [11]

4. SUMMARY

Load distribution along the teeth contact line occurring during the transmission operation is an important factor affecting its durability. Due to the deviation in gear teeth manufacturing, the transmission's body and the elastic distortion of the shafts as a result of inter-tooth force action, it is very difficult to obtain an even load distribution under actual conditions. Any deviations cause an adverse increase in contact stresses. The use of innovative gear designs or eccentric shaft mounting, as described in the article, can contribute to a reduction in the undesirable effects of uneven load distribution and significantly affect the durability and reliability of the transmission. In addition, reducing the unevenness of the load distribution on the width of the tooth also contribute to a reduction in vibrations generated by the transmission and accompanying noise.

The article also presents an example of a structural solution. Despite the relatively low cost of implementing this solution, which is undoubtedly an important advantage, on the basis of the presented analysis, it is stated that, in certain situations at work, there is a collision of elements. For this reason, it is proposed to use this solution in terms of relatively small values of uneven distribution of load on the width of the meshing, rather than treat this solution, for example, as a method for equalizing the load distribution when the transmission's axes are non-parallel.

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