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METHODS OF POSITION ESTIMATION IN PARAMETRIC NAVIGATION

Summary. The estimation of position coordinates of a navigating ship is one of the navigational subprocesses. The methods used in this process are either deterministic (the case of a minimum number of navigational parameters measurements) or probabilistic (in cases where we have access to information redundancy). Naturally, due to the accuracy and reliability of the calculated coordinates, probabilistic methods should be primarily used. The article presents the use of the method of least squares and Kalman filtering in algorithms in integrated navigation for the estimation of position coordinates, taking into account ship movement parameters.
Keywords: navigational data fusion; least squares estimation; Kalman filtering; estimation; navigation; algorithm of integrated navigation.

1. INTRODUCTION

The calculation of ship coordinates involves measurements and calculations of various navigational parameters. The basic navigational parameters include position coordinates and movement (velocity vector). In terms of position determination, there are three groups of navigation methods:

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- Dead reckoning, in which, on the basis of the mathematical model of the ship movement and velocity vector measurements, we can determine a ship's position at any time. Such an approach is known as dead reckoning navigation, with inertial navigation being one of its technical implementations.
- Comparative methods, which involve the comparison of the physical field recorded in an analogue or digital device with its measured values. These methods are used in bathymetric or topographic navigation, that is, navigation based on the measurement of physical fields of the Earth (magnetic, gravitational and other). Today, these methods are also used for comparing radar images with digital charts.
- Parametric methods, using the measurement of physical quantities, which directly or indirectly determine navigational function, i.e., geometric relations between a ship's position and navigational marks' coordinates. This is the primary method of position fixing.

The algorithms for integrated navigation systems involve a fusion of different methods; in particular, the parametric method is combined with the dead reckoning method. This process requires the combined processing of measurement data, which allows us to optimize the use of navigational information. The multisensor fusion of navigational data is widely discussed in the literature, e.g., [7], while GPS data integration with other navigational measurements is described in [3].

These authors present selected variants of the integration of navigational data obtained from different navigation systems. The method of least squares and the classic Kalman filter were used as the mathematical model of measurements integration.

2. THE LEAST SQUARES METHOD

Let us assume that we have measurements of varying accuracy and that we will use the method of least squares with weights for their fusion [9]. In this case, the vector of state (position coordinates) is described by this formula:

$$\mathbf{x} = (\mathbf{G}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{z} \tag{1}$$

and its covariance matrix is written as this relation:

$$\mathbf{P} = (\mathbf{G}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{G})^{-1} \tag{2}$$

where:

x - vector of state
z - vector of measurements
G - a matrix binding the vector of state with the vector of measurements
P - covariance matrix of the state vector
R - covariance matrix of the measurements vector

One of the simple measurement situations is a combination of GPS position coordinates with a *dead reckoning* (*DR*) position. In this case, **G**, the matrix will be a block matrix in the following form:

$$\mathbf{G} = [\mathbf{I}_{2\times 2} \vdots \mathbf{I}_{2\times 2}]^{\mathrm{T}} \tag{3}$$

The matrix of measurements covariance will mean that **R** is also a block matrix:

$$\mathbf{R} = \begin{bmatrix} \mathbf{P}_{GPS} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{DR} \end{bmatrix} \tag{4}$$

where the matrices $\mathbf{P}_{GPS}$ and $\mathbf{P}_{DR}$ are, respectively, matrices of GPS and DR covariances. With these assumptions, the matrix inverse to the covariance matrix of measurements will take this form:

$$\mathbf{R}^{-1} = \begin{bmatrix} \mathbf{P}_{GPS}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{DR}^{-1} \end{bmatrix} \tag{5}$$

Here, we have used the matrix inversion by division into blocks [8]:

$$\mathbf{A} = \begin{bmatrix} \mathbf{P} & \mathbf{Q} \\ \mathbf{R} & \mathbf{S} \end{bmatrix} \text{to} \mathbf{A}^{-1} = \begin{bmatrix} \mathbf{K} & \mathbf{L} \\ \mathbf{M} & \mathbf{N} \end{bmatrix}$$

where:

$$\mathbf{N} = (\mathbf{S} - \mathbf{R}\mathbf{P}^{-1}\mathbf{Q})^{-1}, \qquad \mathbf{L} = -\mathbf{P}^{-1}\mathbf{Q}\mathbf{N}$$
$$\mathbf{M} = -\mathbf{N}\mathbf{R}\mathbf{P}^{-1}, \qquad \mathbf{K} = \mathbf{P}^{-1}(\mathbf{I} - \mathbf{Q}\mathbf{M})$$

Ultimately, with the above assumptions, we get the vector of state (position coordinates):

$$\mathbf{x} = (\mathbf{P}_{GPS}^{-1} + \mathbf{P}_{DR}^{-1})^{-1}[\mathbf{P}_{GPS}^{-1} \vdots \mathbf{P}_{DR}^{-1}]\mathbf{z} \tag{6}$$

and its covariance matrix:

$$\mathbf{P} = (\mathbf{P}_{GPS}^{-1} + \mathbf{P}_{DR}^{-1})^{-1} \tag{7}$$

3. KALMAN FILTER

Kalman filtering is commonly used today [5], [6]. It is implemented at various levels of navigational information processing, from physical measurements by sensors (preliminary processing), through the combination of measurements from different sensors (intermediate processing) to the estimation of position coordinates and other navigational parameters (final processing). At each of these levels, we use the same mathematical tools and the same computing algorithm.

The discrete Kalman filter, in a particular case, describes the system of two equations [1], [2], [5], [6]:

- State equation (structural model)

$$\mathbf{x}_{i+1} = \mathbf{A}_{i+1,i}\mathbf{x}_i + \mathbf{w}_i \tag{8}$$

- Measurement equation (measurement model)

$$\mathbf{z}_{i+1} = \mathbf{C}_{i+1}\mathbf{x}_i + \mathbf{v}_i \tag{9}$$

where:

x - *n*-dimensional vector of state
w - *r*-dimensional vector of state disturbances
z - *m*-dimensional vector of measurements
v - *p*-dimensional vector of measurement disturbances (identified with measurement noise)
A - *n*’*n*-dimensional transition matri
C - *m*’*n*-dimensional measurement matrix
$r \le n$, $p \le m$.

We assume that the vectors of disturbances **w** and **v** are Gaussian noise, with normal distribution and a zero mean vector, and are mutually non-correlated. In the case of colour noise (with a trend), the extended Kalman filter is applied, where the disturbance trend is included as additional components of the state vector.

The equation of state describes the evolution of the dynamic system described in the state space, while the model of measurements functionally combines measurements with the system state. The solution to Equations (8) and (9), taking into account the constraints imposed on the vectors of disturbances, is the Kalman filter. Calculation of the state vector in the Kalman filter is described by the following algorithm:

- Projection of the state vector:

$$\tilde{\mathbf{x}}_{\mathrm{i+1,1}} = \mathbf{A}_{\mathrm{i+1,i}}\hat{\mathbf{x}}_{\mathrm{i}} \tag{10}$$

where $\tilde{\mathbf{x}}$ is the projected value of the state vector, and $\hat{\mathbf{x}}$ is the estimated value of the state vector

- Covariance matrix of the projected state vector:

$$\mathbf{P}_{i+1,i} = \mathbf{A}_{i+1,i}\mathbf{P}_i\mathbf{A}_{i+1,1}^{\mathrm{T}} + \mathbf{Q}_i \tag{11}$$

where **Q** is the covariance matrix of disturbances of the state (of vector **w**)

- Innovation process:

$$\boldsymbol{\varepsilon}_{i+1} = \mathbf{z}_{i+1} - \mathbf{C}_{i+1}\tilde{\mathbf{x}}_{i+1,i} \tag{12}$$

- Covariance matrix of the innovation process:

$$\mathbf{S}_{i+1} = \mathbf{R}_{i+1} + \mathbf{C}_{i+1}\mathbf{P}_i\mathbf{C}_{i+1}^{\mathrm{T}} \tag{13}$$

where **R** is the covariance matrix of measurement disturbances (of vector **v**)

- Filter gain matrix (Kalman matrix):

$$\mathbf{K}_{i+1} = \mathbf{P}_{i+1,1}\mathbf{C}_{i+1}^{\mathrm{T}}\mathbf{S}_{i+1}^{-1} \tag{14}$$

- Estimated value of the state vector from filtering after measurement $\mathbf{z}_{i+1}$:

$$\hat{\mathbf{x}}_{i+1} = \tilde{\mathbf{x}}_{i+1,i} + \mathbf{K}_{i+1}\boldsymbol{\varepsilon}_{i+1} \tag{15}$$

- Covariance matrix of the estimated state vector:

$$\mathbf{P}_{i+1} = (\mathbf{I} - \mathbf{K}_{i+1}\mathbf{C}_{i+1})\mathbf{P}_{i+1,i} \tag{16}$$

4. THE STRUCTURE OF THE INTEGRATING FILTER

The adopted mathematical model of ship movement and the configuration of navigational devices affect the structure of the Kalman filter algorithm. Let us assume, as in the position estimation algorithm by the method of least squares, that position coordinates are determined using GPS (parametric navigation), while measurements in dead reckoning navigation are obtained from a gyroscopic compass and Doppler log.

Let us define the state vector as:

$$\mathbf{x} = [\varphi, \lambda, V_N, V_E, COG, SOG]^{\mathrm{T}} \tag{17}$$

where:

φ	-	latitude
λ	-	longitude
V_N, V_E	-	projections of the vector of speed over ground on the parallel and the meridian
COG	-	course over ground
SOG	-	speed over ground

The transition matrix **A** of the structural model will be:

$$\boldsymbol{A}_{i+1,i}=\begin{bmatrix}1 & 0 & k_\varphi\cdot\Delta t_i & 0 & 0 & 0\\ 0 & 1 & 0 & k_\lambda\cdot\Delta t_i & 0 & 0\\ 0 & 0 & 1+\Delta V_N & 0 & 0 & 0\\ 0 & 0 & 0 & 1+\Delta V_E & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix} \tag{18}$$

where:

$$\Delta V_N=\frac{\bar{V}_N-V_N}{V_N},\ \Delta V_E=\frac{\bar{V}_E-V_E}{V_E} \tag{19}$$

$$k_\varphi=\frac{\sqrt{(1-e^2\sin^2\varphi)^3}}{a(1-e^2)},\ k_\lambda=\frac{\sqrt{1-e^2\sin^2\varphi}}{a\cos\varphi} \tag{20}$$

a - major semi-axis of the Earth's ellipsoid
e - the first eccentricity of the Earth's ellipsoid
$\bar{V}_E$i $\bar{V}_N$ - vectors of mean velocity, along the parallel and the meridian, respectively
k_φ, k_λ - coefficients of conversion of angular measure to linear measure on the reference ellipse, on the meridian and parallel, respectively, dependent on the latitude of the ship's position and reference ellipsoid parameters (in this case, WGS-84)

Another element of the structural model is the covariance matrix of the state disturbance vector **Q**. Its elements define a priori distributions of disturbances of the estimated quantities. For the state vector, as defined by Formula (10), the matrix of the state **Q** disturbances may assume this form:

$$\boldsymbol{Q}_i=\begin{bmatrix}q_{11} & q_{12} & 0 & 0 & 0 & 0\\ q_{21} & q_{22} & 0 & 0 & 0 & 0\\ 0 & 0 & q_{33} & q_{34} & 0 & 0\\ 0 & 0 & q_{43} & q_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & q_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & q_{66}\end{bmatrix} \tag{21}$$

where:

$q_{11}=k_\varphi^2\left(\sigma_\varphi^2+\Delta t_i^2\sigma_{V_N}^2\right)$,
$q_{22}=k_\lambda^2\left(\sigma_\lambda^2+\Delta t_i^2\sigma_{V_E}^2\right)$,
$q_{12}=q_{21}=k_\varphi k_\lambda\Delta t_i^2\sigma_{V_NV_E}$,
$q_{33}=\sigma_{V_N}^2$,
$q_{44}=\sigma_{V_E}^2$,
$q_{34}=q_{43}=\sigma_{V_NV_E}$,
$q_{55}=\sigma_{\Delta COG}^2$,
$q_{66}=\sigma_{\Delta SOG}^2$,
σ_φ - disturbance of the ship's movement along the latitude (yawing)
σ_λ - disturbance of the ship's movement along the longitude (yawing)

$$\sigma_{V_N}^2=\ [(\sigma_{SOG}\cos COG)^2+(SOG\sigma_{COG}\sin COG)^2] \tag{22}$$

$$\sigma_{V_E}^2=\ [(\sigma_{SOG}\sin COG)^2+(SOG\sigma_{COG}\cos COG)^2] \tag{23}$$

$$\sigma_{V_NV_E}=\frac{1}{2}(\sigma_{SOG}^2-SOG^2\sigma_{COG}^2)\sin 2COG \tag{24}$$

σ_{COG} - *COG* measurement error
σ_{SOG} - *SOG* measurement error

The quantities measured (measurement model) are the following parameters: position coordinates from a GPS system$(\varphi_{GPS}, \lambda_{GPS})$, projections of the vector of speed over ground on the parallel and meridian (V_N, V_E), course over ground (*COG*) and speed over ground (*SOG*). Hence, the vector of measurements will take this form:

$$\mathbf{z} = [\varphi_{GPS}, \lambda_{GPS}, V_N, V_E, COG, SOG]^{\mathbf{T}} \tag{25}$$

The matrix of measurements is the Jacobi matrix, which has the following form:

$$C = \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & B_1 & B_2 & -1 & 0 \\ 0 & 0 & E_1 & E_2 & E_3 & -1 \end{bmatrix} \tag{26}$$

$$B_1 = \begin{cases} \dfrac{COG}{2V_N}, & V_N \neq 0, \\ \dfrac{COG}{2}, & V_N = 0, \end{cases}$$

$$B_2 = \begin{cases} \dfrac{COG}{2V_E}, & V_E \neq 0, \\ \dfrac{COG}{2}, & V_E = 0, \end{cases}$$

$$E_1 = \left(1 - \frac{V_E^2}{V}\right) \cos COG + \frac{V_N V_E}{V} \sin COG,$$

$$E_2 = \left(1 - \frac{V_N^2}{V}\right) \sin COG + \frac{V_N V_E}{V} \cos COG,$$

$$E_3 = V_N \sin COG - V_E \sin COG$$

$$V = \sqrt{V_N^2 + V_E^2}$$

The matrix of measurement disturbance covariance (measurement vector) is also an element of the measurement model. It is a band matrix because certain quantities measured are not correlated with each other, e.g., GPS measurements and components of speed from dead reckoning navigation, or gyroscope and log measurements.

$$\mathbf{R} = \begin{bmatrix} \sigma_\varphi^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_\lambda^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{V_E}^2 & \sigma_{V_E V_N} & 0 & 0 \\ 0 & 0 & \sigma_{V_E V_N} & \sigma_{V_N}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{COG}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{SOG}^2 \end{bmatrix} \tag{27}$$

5. SUMMARY

The presented models and algorithms illustrate two of many possibilities regarding the navigational application of the method of least squares and the Kalman filter for the integration of navigational data. In the Kalman filter, the state vector reproduces system evolution (movement trend) on the basis of dead reckoning navigation. The main advantages of the Kalman filter, in this case, are its recurrence, which is a natural necessity in case of ship navigation, and the possibility of using the ship movement data (its trend).

There are other approaches to Kalman filtering, based on Monte Carlo simulations [4] and artificial intelligence methods [10], which enable identification of the state model parameters and *online* measurements.

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