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SIMULATION OF VERTICAL VEHICLE NON-STATIONARY RANDOM VIBRATIONS CONSIDERING VARIOUS SPEEDS

Summary. The aim of the paper is the application of evolutionary non-stationary random vibration theory in the classic statistical solution vibrations. The dynamic model parameters are the deterministic function. The evolutionary non-stationary random function will be modelled by changeable speed of the vehicle model and vertical irregularity of track. It'll assume evolutionary Gaussian process.
Keywords: non-stationary random process, mean response, covariance response.

SYMULACJA PIONOWYCH NIESTACJONARNYCH PRZYPADKOWYCH WIBRACJI POJAZDU Z PRĘDKOŚCIĄ ZMIENNĄ

Streszczenie. Celem artykułu jest zastosowanie teorii ewolucyjnych niestacjonarnych przypadkowych procesów w rozwiązywaniu problemu drgania pojazdów z prędkością zmienną. Model dynamiczny pojazdu zakłada parametry deterministyczne, a niestacjonarność procesu będzie modelowana właśnie za pomocą zmiennej prędkości oraz pionowych nierówności trasy. Będzie przy tym uwzględniany proces ewolucyjny Gaussa.
Słowa kluczowe: niestacjonarny proces przypadkowy, wartość średnia, kowariancja.

1. INTRODUCTION

A lot of papers in stochastic dynamics are devoted to Gaussian stationary excitations but only a few random processes in engineering practice are really Gaussian and stationary. Stochastic loadings will be interpreted not only as external forces, but also as external kinematic effects. Bolotin defined [1] random excitation as follows loading due to atmospheric turbulence, acoustic loading, loading due to pulsation in a turbulent boundary layer, loading due to pressure of sea waves, loading of transport machines due to unevenness of track, and seismic loading.

In the case of stochastic systems (especially non-linear) we encounter the approaches, such as: tangent linearization method (TLM) [8], statistical linearization method (SLM) in various modifications [1, 2, 3, 5, 7], statistical quadratization method, the Markov process approach (MPT) [9], functional method of Volterra and Wiener (FMVW) [8], asymptotic

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method of Krylov–Bogoljubov–Mitropolsky (ASM) [2], perturbation method (PM) and its modifications [2, 6, 7].

Thanks to computer techniques, Monte Carlo simulation method (MCS) is very popular and frequently applied [8, 9]. Although this method is straightforward and does not have such limitations, is generally time-consuming and costly. In view of these difficulties, approximate methods, including PM, TLM, SLM, SQM can be advantageous. Some authors look for the new approaches of the solution by combining the Monte Carlo method with other methods [4].

2. MATHEMATICAL MODEL

To construct a mathematical model of a system for dynamic analysis, it is necessary to idealize the inertia, damping and stiffness properties by discrete or continuous elements. Usually the first step is to construct a physical model that may be an assemblage of discrete elements such as mass, springs and dashpots, continuous elements such as bars, beams, shells and volumes, or a combination of both discrete and continuous elements. The application of the fundamental laws of mechanics yields a set of generally non-linear differential equations

$$\dot{\mathbf{x}}(t) + \mathbf{A}(\mathbf{x},t)\cdot\mathbf{x}(t) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{x}(t)$ is the response vector corresponding to the random excitation vector $\mathbf{f}(t)$, $\mathbf{A}(\boldsymbol{x},t)$ is the real or complex structural matrix of order $n\times n$. $\mathbf{A}(\boldsymbol{x},t)$ may be linear or non-linear, depending on the nature of the problem [10]. Many mechanical models are linear thanks to their analytical simplicity and the fact that they yield realistic results for large class problems.

There are, however, a number of problems for which linear models do not yield acceptable results, so that it becomes necessary to construct non-linear models. It means, if $\mathbf{A}(\boldsymbol{x},t)$ is non-linear, we can apply well-known approximate methods (PM, TLM, SLM or SQM). Using linearization techniques we get the statistically equivalent structural matrix $\boldsymbol{A}$.

This study presents two approaches in determining the response of a system modelled by equation (1) by the Markov processes theory; and Monte Carlo simulation.

The Markov process formulation requires the idealization that the excitation is independent at two instants of time regardless of how close they are (delta correlation) [9]. This assumption, which is clearly physically unrealizable, leads to such models as white noise and processes obtained by linearly filtering white noise.

Let us consider the system of first-order differential equations (1) with initial conditions $\mathbf{x}(0) = \mathbf{0}$ and force excitation $\mathbf{f}(t) = \mathbf{y}(t).p(t)$. The force $\mathbf{f}(t)$ is a modulated evolutionary process vector with a deterministic vector function $\mathbf{y}(t)$ and stationary random process $p(t)$ with zero mean. The mean response of $\mathbf{x}(t)$ is

$$\dot{\mathbf{m}}_x(t) + \mathbf{A}\cdot\mathbf{m}_x(t) = \mathbf{m}_f(t)\,, \tag{2}$$

where $\mathbf{m}_x = E[\mathbf{x}]$ is the mean vector and $E[\ldots]$ is the value operator. The covariance response of $\mathbf{x}(t)$ is

$$\dot{\mathbf{K}}(t) + \mathbf{A}\cdot\mathbf{K}(t) + (\mathbf{A}\cdot\mathbf{K}(t))^T = \mathbf{b}\cdot\mathbf{y}^T + \mathbf{y}\cdot\mathbf{b}^T\,, \tag{3}$$

where $\mathbf{K}(t)$ is the covariance matrix,

$$\mathbf{K}(t) = E[\mathbf{x}_c \cdot \mathbf{x}_c^T] \quad \text{and} \quad \mathbf{K}(0) = \mathbf{0}, \tag{4}$$

and

$$\mathbf{b}(t) = \mathbf{h}(t) \cdot \int_0^t \mathbf{h}^{-1}(u) \cdot \mathbf{y}(u) \cdot E[p(u) \cdot p(t)] \cdot du \,. \tag{5}$$

Equations (2) and (3) imply that mean vector and covariance matrix are the time functions. Matrix $\mathbf{h}(t)$ is so-called the fundamental solution matrix or impulse response matrix. If it is assumed that $p(t)$ is white noise with $E[p(t_1).p(t_2)] = 2.\pi.\Phi_0.\delta(t_2\text{-}t_1)$, then the equation (3) can be expressed as

$$\dot{\mathbf{K}}(t) + \mathbf{A} \cdot \mathbf{K}(t) + (\mathbf{A} \cdot \mathbf{K}(t))^T = 2 \cdot \pi \cdot \Phi_0 \cdot \mathbf{y}(t) \cdot \mathbf{y}^T(t), \tag{6}$$

where Φ_0 is the power spectral density of $p(t)$. The acceptable solution of the equation (6) is possible to make by special numerical approach.

Let us consider Crank-Nicolson integration method. The discrete time derivation is given by

$$\dot{\mathbf{m}}_x(t) = \frac{2}{\Delta} \cdot [\mathbf{m}_x(t) - \mathbf{m}_x(t-\Delta)] - \dot{\mathbf{m}}_x(t-\Delta) \quad \text{and} \quad \dot{\mathbf{K}}(t) = \frac{2}{\Delta} \cdot [\mathbf{K}(t) - \mathbf{K}(t-\Delta)] - \dot{\mathbf{K}}(t-\Delta), \tag{7a,b}$$

where Δ is the time step of the integration method. Using the equations (2), (6) and (7) we can write

$$(\frac{2}{\Delta} \cdot \mathbf{I} + \mathbf{A}) \cdot \mathbf{m}_x(t) = \mathbf{m}_f(t) + \frac{2}{\Delta} \cdot \mathbf{m}_x(t-\Delta) + \dot{\mathbf{m}}_x(t-\Delta), \tag{8}$$

$$(\frac{2}{\Delta} \cdot \mathbf{I} + \mathbf{A}) \cdot \mathbf{K}(t) + [\mathbf{A} \cdot \mathbf{K}(t)]^T = 2 \cdot \pi \cdot \Phi_0 \cdot \mathbf{y}(t) \cdot \mathbf{y}^T(t) + \frac{2}{\Delta} \cdot \mathbf{K}(t-\Delta) + \dot{\mathbf{K}}(t-\Delta), \tag{9}$$

where $\mathbf{I}$ is the identity matrix. The equation (9) is so-called Lyapunov-Sylvester equation subjected to Crank-Nicolson integration approach. In each time step is necessary to use the special numerical algorithm assembled and applied in MATLAB.

3. TESTING ANALYSIS - RAILWAY VEHICLE VIBRATION

A vehicle moving on railway track causes vibrations. Since the profile of a track is a random function of the spatial coordinates, these vibrations are also random. We shall assume that the motion of the vehicle in the horizontal direction is non-uniform (changeable speed, although more important non-stationarity can be the changeable track quality).

Using previous theory we shall solve the response of the simple vehicle model (Fig. 1) under non-stationary random excitation. Let us determine the first and second statistical moments (i.e. the mean vector and the covariance matrix) of the response of the mechanical model on Fig.1. The structural parameters are: mass of

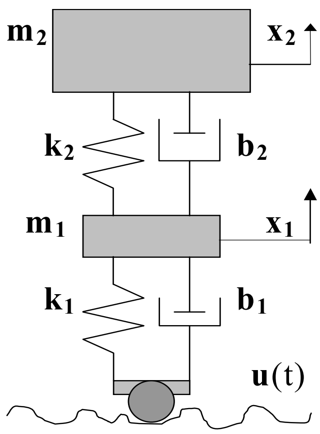

Fig. 1. Dynamic model of vehicle
Rys. 1. Dynamiczny model pojazdu

bogie m_1*=3000 kg,* mass of body of coach m_2*=13000 kg,* damping coefficient in vertical direction
b_1*=120000 Nsm*$^{-1}$, damping coefficient in vertical direction b_2*=100000 Nsm*$^{-1}$, vertical stiffness k_1*=3000000 Nm*$^{-1}$, vertical stiffness k_2*=1500000 Nm*$^{-1}$.

Let us consider the approximation of the power spectral density of vertical unevenness $u(t)$ of track in due order ORE B 176 in the form

$$S_{uu}(\lambda)=\frac{A\cdot b^2}{(\lambda^2+a^2)\cdot(\lambda^2+b^2)}, \tag{10}$$

where a = 0.0206, b=0.8246, A = $4.032\cdot 10^{-7}$ for a good track and A = $1.08\cdot 10^{-8}$ for a bad track. λ is the length frequency. If the vehicle speed is time function $v = v(t)$ and $\lambda=\frac{\omega}{v}$, then

$$S_{uu}(\omega,t)=\frac{1}{v(t)}\cdot\frac{A\cdot b^2}{[\frac{\omega^2}{v^2(t)}+a^2]\cdot[\frac{\omega^2}{v^2(t)}+b^2]}, \tag{11}$$

where ω is the circular frequency.

Applying the Markov process theory we shall need to use the assumption of an evolutionary random excitation with a deterministic modulated function and white noise process. Therefore, it is necessary to define the filter parameters of the excitation function. A commonly used filter in modelling of the earthquake ground motion is the Kanai-Tajimi filter governed by the following differential equation

$$m_e\cdot\ddot{u}+b_e\cdot\dot{u}+k_e\cdot u=w(t), \tag{12}$$

where $w(t)$ is well-known Gaussian white noise process with constant power spectral density S_0. The frequency response function of the filter can be expressed as

$$H(\omega)=\frac{1}{k_e-\omega^2\cdot m_e+i\cdot\omega\cdot b_e}, \tag{13}$$

Comparing the power spectral density of *u(t)* from (12) and (11) we get

$$\frac{S_0}{(k_e-\omega^2\cdot m_e)+b_e^2\cdot\omega^2}=\frac{1}{v}\cdot\frac{A\cdot b^2}{\left[\frac{\omega^2}{v^2}+a^2\right]\cdot\left[\frac{\omega^2}{v^2}+b^2\right]}, \tag{14}$$

From (14) it is clear that $S_0=A\cdot b^2$, $m_e=\frac{1}{\sqrt{v^3},}$ $b_e=\frac{(a+b)^2}{v}$, $k_e=a\cdot b\cdot\sqrt{v}$.

Let us construct the equations of motion

$$\begin{bmatrix} m_1 & 0\\ 0 & m_2\end{bmatrix}\cdot\begin{Bmatrix}\ddot{x}_1\\ \ddot{x}_2\end{Bmatrix}+\begin{bmatrix} b_1+b_2 & -b_2\\ -b_2 & b_2\end{bmatrix}\cdot\begin{Bmatrix}\dot{x}_1\\ \dot{x}_2\end{Bmatrix}+\begin{bmatrix} k_1+k_2 & -k_2\\ -k_2 & k_2\end{bmatrix}\cdot\begin{Bmatrix}x_1\\ x_2\end{Bmatrix}=\begin{bmatrix} b_1 & 0\\ 0 & 0\end{bmatrix}\cdot\begin{Bmatrix}\dot{u}\\ 0\end{Bmatrix}+\begin{bmatrix} k_1 & 0\\ 0 & 0\end{bmatrix}\cdot\begin{Bmatrix}u\\ 0\end{Bmatrix}. \tag{15}$$

Considering (11), (13) and 2-dimensional state vector by substitute $y_1=x_1,\ y_2=x_2,\ y_3=u,\ y_4=\dot{x}_1,\ y_5=\dot{x}_2,\ y_6=\dot{u}$ the equations of motion can be expressed as

$$\dot{\mathbf{y}}(t)=\mathbf{A}(t)\cdot\mathbf{y}(t)+\mathbf{b}(t)\cdot w(t), \tag{16}$$

$$\begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \\ \dot{y}_5 \\ \dot{y}_6 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\left(\frac{k_1+k_2}{m_1}\right) & \frac{k_2}{m_1} & \frac{k_1}{m_1} & -\left(\frac{b_1+b_2}{m_1}\right) & \frac{b_2}{m_1} & \frac{b_1}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & 0 & \frac{b_2}{m_2} & -\frac{b_2}{m_2} & 0 \\ 0 & 0 & -a \cdot b \cdot v(t) & 0 & 0 & (a+b)^2 \cdot \sqrt{v(t)} \end{bmatrix} \cdot \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sqrt{v^3(t)} \end{Bmatrix} \cdot w(t). \tag{17}$$

Considering $E(\mathbf{y}) = \mathbf{0}$ we obtain the covariance response by using (4) as follows

$$\dot{E}[\mathbf{y} \cdot \mathbf{y}^T] + \mathbf{A} \cdot E[\mathbf{y} \cdot \mathbf{y}^T] + (\mathbf{A} \cdot E[\mathbf{y} \cdot \mathbf{y}^T])^T = 2 \cdot \pi \cdot S_0 \cdot \mathbf{b}(t) \cdot \mathbf{b}^T(t), \tag{18}$$

The numerical solution can be realizing by (9).

If $v(t) = \frac{100}{3,6} \cdot \left[1 + 0,4 \cdot \sin\left(\frac{\pi \cdot t}{T}\right)\right]$, then the time modulation function is

$$\mathbf{b}^T(t) = \left[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \sqrt{\left(\frac{100}{3,6} \cdot \left[1 + 0,4 \cdot \sin\left(\frac{\pi \cdot t}{T}\right)\right]\right)^3} \right], \tag{19}$$

where T (= 120 [s]) is duration of the simulation. The results of the solution are shown in graphic form on Figs. 2 - 4. We compare the standard deviation of vertical displacements, velocities and accelerations of mass bodies 1 and 2 for the track quality parameter $A = 4.032 \cdot 10^{-7}$ (good track) and $A = 1.08 \cdot 10^{-8}$ (bad track).

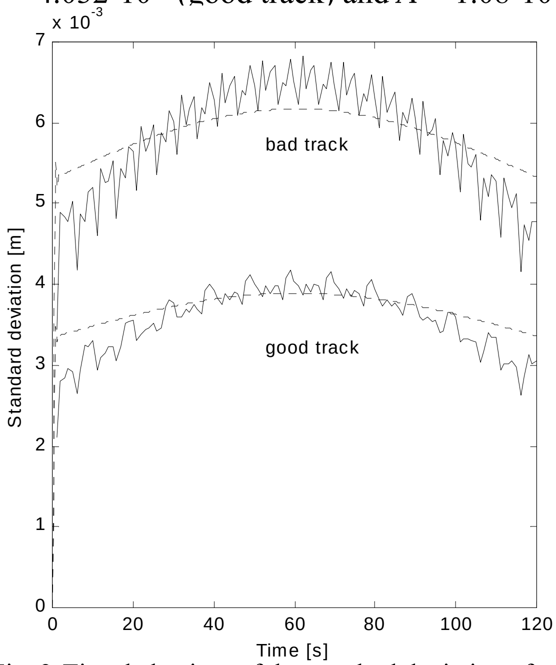

Fig. 2. Time behaviour of the standard deviation of displacement x_1
——— Monte Carlo simulation
············ Markov process theory

Rys. 2. Charakterystyka czasowa odchylenia standardowego
——— symulacja Monte Carlo
············ symulacja procesu Markowa

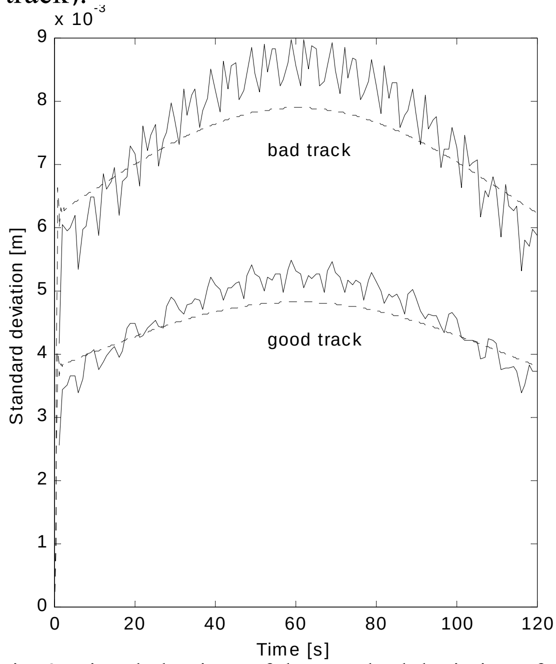

Fig. 3. Time behaviour of the standard deviation of displacement x_2
——— Monte Carlo simulation
············ Markov process theory

Rys. 3. Charakterystyka czasowa odchylenia standardowego
——— symulacja Monte Carlo
············ symulacja procesu Markowa

4. CONCLUSION

In our study, a non-stationary vibration description is extended to the dynamic analyses of vehicles by using the Markov process theory and "classic" Monte Carlo approach, which eliminate the traditional restriction of constant speed (or the track quality) during the period oscillation. Particularly, after a series of numerical analyses (Monte Carlo), the presented Markov vector approach is very effective and rapid with respect to the computational time (approximately fifty times more rapid). The Monte Carlo simulation is applied to check the accuracy of the results, which show a fairly good comparison. Finally, it should be emphasized that these statistically responses are very useful for estimating the reliability of the vehicles structures.

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