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IMPLEMENTATION OF DISCRETE FULLY STRESSING INTO STRUCTURAL OPTIMIZATION

Summary. The paper presents numerical study of the discrete formulation of the fully stress design (FSD) algorithm in the case of the thin shell finite elements. The goal will be to present interesting original mathematical description of the direct strength designing. Using numerical tests the effectiveness of the proposed algorithms will be compared.
Keywords. Stress analysis, thin shell finite element, fully stress design.

IMPLEMENTACJA DYSKRETNEGO WARTOŚCIOWANIA DLA STANU PEŁNEGO NAPRĘŻENIA W CELU OPTYMALIZACJI KONSTRUKCJI

Streszczenie. Artykuł jest poświęcony numerycznemu studium dyskretnej formuły algorytmu projektowania dla stanu pełnego naprężenia (FSD algoritmus) w przypadku cienkich skorupowych elementów skończonych. Celem będzie zaprezentowanie interesującego oryginalnego matematycznego opisu bezpośredniego projektowania wytrzymałościowego. Efektywność zaproponowanych algorytmów zostanie porównana za pomocą testów numerycznych.
Słowa kluczowe. Analiza naprężenia, cienki skorupowy element skończony, projektowanie dla stanu pełnego naprężenia.

1. INTRODUCTION

Expansion of computational technique allowed putting qualitatively new approaches in designing machines and appliances into practice. The problem of proper designing and constructing of machines gets new dimensions and wide scope for solving other unsolved problems by establishing computers and consequent creating and developing corresponding software. An optimized design is comprehended as a technically realizable design of structure which is the best from all possible designs for a given goal [3].

Optimization of mechanical systems combines numerical mathematics and engineering mechanics. It is used in applications in civil engineering, mechanical engineering, automotive and ship-building industry, and so on. It made the biggest progress in last thirty years thanks to utilizing very fast numerical computers and computer graphics. When choosing cost,

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weight of structure or maximum power at limited cost as a design criterion, the importance of optimization is evident.

First formulations of optimization problems in form of mathematical programming were occurring approximately since 1960. One of the pioneers, who significantly influenced the development of optimal designing of constructions of machines and their components, was undoubtedly Schmit. He linked optimization methods with a new and progressive computational method at that time - finite element method as one of the first. At that time, the weight of monitored object or some strength condition was the objective function. Optimization process was gradually improving by adding other limiting conditions. In the second half of last century other works of similar nature, which extended options in the field of optimal designing of parameters of machines and their components into automated approaches occurred. We cannot omit works of Kirch, Morrow, or Gallagher. There were designed plenty of effective approaches based not only on purely mathematical comprehension of optimization problem, but also a little bit non-traditional or more precisely unaccustomed approaches which play an important role in solving various technical problems. These approaches use some of the basic principles of mechanics. For example, the method which is known as fully stress design (FSD) originated from the idea of independence of axial forces in statically determinate truss structures. Its application is useful, mainly thanks to its effectiveness. However, it is limited only to problems of strength dimensioning and it turned out to be certain disadvantage in creating universal program systems. In this article theory of FSD will be described and applied specially for truss, beam and shell structures in spite of its lower universality [3, 6].

2. FULLY STRESS DESIGN ALGORITHMIZATION IN THE CASE OF THIN SHELL FINITE ELEMENT

We will focus on well-known shell finite elements (Kirchhoff's or Mindlin's formulation) [8, 10, 11, 12], mainly on the stress computation. The stiffness parameters depend on material constants and element geometry, mainly on its thickness. The stress computation process is based on the expression of the *j*-th element membrane forces and bending moments (without shear forces) [10, 9], i.e.

$$[F_{xx} \quad F_{yy} \quad F_{xy}]_j^T = \boldsymbol{F}_m^j = \int_S \boldsymbol{E}_m^j \cdot \boldsymbol{\varepsilon}_m^j \, dS_j = \boldsymbol{E}_m^j \cdot \int_S \boldsymbol{B}_m^j \cdot dS_j \cdot \boldsymbol{u}_L^j = t_j \cdot \boldsymbol{D}_j \cdot \boldsymbol{I}_m^j \cdot \boldsymbol{u}_L^j \quad (1)$$

and

$$[M_{xx} \quad M_{yy} \quad M_{xy}]_j^T = \boldsymbol{M}_b^j = \int_S \boldsymbol{E}_b^j \cdot \boldsymbol{\varepsilon}_b^j \, dS_j = \boldsymbol{E}_b^j \cdot \int_S \boldsymbol{B}_b^j \cdot dS_j \cdot \boldsymbol{u}_L^j = \frac{t_j^3}{12} \cdot \boldsymbol{D}_j \cdot \boldsymbol{I}_b^j \cdot \boldsymbol{u}_L^j . \quad (2)$$

The auxiliary matrices $\mathbf{I}_m$ and $\mathbf{I}_b$can be calculated only using the numerical approach. Further details about $\mathbf{E}_m$,$\mathbf{E}_b$, $\mathbf{D}$, $\mathbf{B}_m$, $\mathbf{B}_b$, $\mathbf{u}_{el}$ and t are presented in [9]. The extreme stress values can be expected at the top or at the bottom surface. We will deal with 3 basic types of algorithms based on the FSD.

2.1. Algorithm no. 1 - classic FSD

Classic FSD algorithm assumes linear relation between stress in shell and inverse value of shell thickness. This assumption arises from the assumption of constant force quantities whose values does not depend on shell thickness. The algorithm arises from the similarity of the triangles 0AB and 0CD as can be seen in Fig. 1. This similarity can be expressed by relation 3.

$$\frac{\sigma_{ieq}^{(k)}}{\left(\frac{1}{t_i}\right)^{(k)}} = \frac{\sigma_L}{\left(\frac{1}{t_i}\right)^{(k+1)}}. \tag{3}$$

By means of this equation it is possible to derive the relation for the calculation of new value of optimizing variable (new thickness)

$$t_i^{(k+1)} = \frac{\sigma_{ieq}^{(k)} . t_i^{(k)}}{\sigma_L}. \tag{4}$$

The geometric interpretation shows, that new estimation $t_i^{(k+1)}$is preformed from the points [0,0] and [$\left(\frac{1}{t_i}\right)^{(k)}$,$\sigma_{ieq}^{(k)}$].From the numerical mathematics point of view we are speaking about RegulaFalsi method (method of chords-secants). For each optimizing group maximum stress and thickness value from the previous iteration step are introduced into equation (4) and value $t_i^{(k+1)}$ is calculated. This value is compared with values from the vector of possible values of the design variables. The nearest higher value is chose as a new value of the optimizing variable.

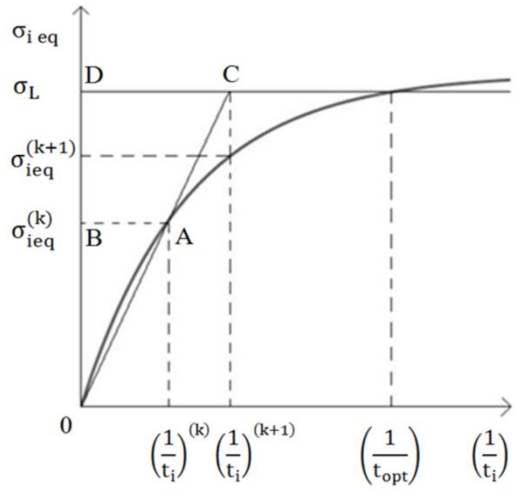

Fig. 1. Geometric interpretation of the algorithm no.1
Rys. 1. Geometryczna interpretacja algorytmu nr 1

2.2. Algorithm no. 2 - approximated FSD

It arises from Newton's tangent method (also known as the Newton-Raphson method). Its problem is to compute the derivative of stress with respect to the design variable.This computation will be realised numerically. Then the effective formulation for the FSD can be derived from the geometry shown in Fig. 2 from the similarity of the traingles ABC and BDE, i.e.

$$\frac{\sigma_{ieq}^{(k)}-\sigma_{ieq}^{(k-1)}}{\left(\frac{1}{t_i}\right)^{(k)}-\left(\frac{1}{t_i}\right)^{(k-1)}} = \frac{\sigma_L-\sigma_{ieq}^{(k)}}{\left(\frac{1}{t_i}\right)^{(k+1)}-\left(\frac{1}{t_i}\right)^{(k)}}. \tag{5}$$

The inverse value of new design variable will be

$$\left(\frac{1}{t_i}\right)^{(k+1)} = \left(\frac{1}{t_i}\right)^{(k)} + \frac{\sigma_L-\sigma_{ieq}^{(k)}}{\sigma_{ieq}^{(k)}-\sigma_{ieq}^{(k-1)}}\left[\left(\frac{1}{t_i}\right)^{(k)} - \left(\frac{1}{t_i}\right)^{(k-1)}\right]. \tag{6}$$

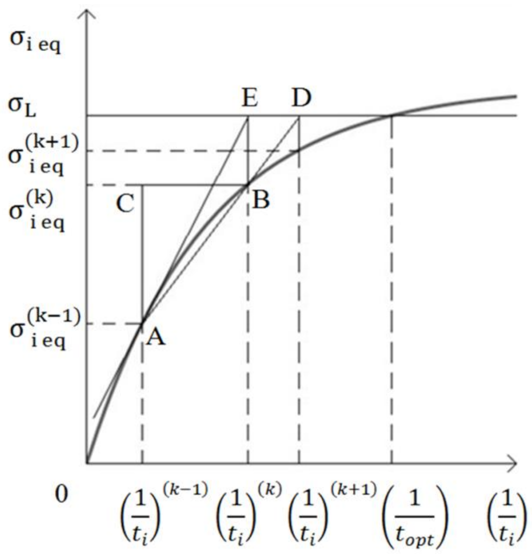

Fig. 2. Geometric interpretation of the algorithm no.2
Rys. 2. Geometryczna interpretacja algorytmu nr 2

Procedure of the computation for the 1st iteration step

For the calculation of new value of the optimizing variable it is necessary to know the stress values from two previous iteration steps. Therefore it is necessary to perform the computation of new optimizing variable in the 1st iteration step by another algorithm. For that reason the 1st iteration step in testing problems where the algorithm no. 2 was utilized was always solved by means of the algorithm no.3.

Procedure of choosing new value of the optimizing variable

After introducing the known values into the equation

$$(t_i)^{(k+1)} = \frac{1}{\left(\frac{1}{t_i}\right)^{(k)} + \frac{\sigma_L - \sigma_{ieq}^{(k)}}{\sigma_{ieq}^{(k)} - \sigma_{ieq}^{(k-1)}}\left[\left(\frac{1}{t_i}\right)^{(k)} - \left(\frac{1}{t_i}\right)^{(k-1)}\right]}. \tag{7}$$

The value $(t_i)^{(k+1)}$ is calculated and consequently it is compared with values from the vector of possible values of the design variables. The nearest higher value is chose as a new value of the optimizing variable.

2.3. Algorithm no. 3 - FSD with linear approximation of force quantities

Numerical testing showed that the assumption of constant force quantities in the classic interpretation of the FSD (the algorithm no. 1) does not respond to real state. Advantage of the algorithm no. 3 in comparison with the classic version of the FSD is the fact, that it assumes linear approximation of the internal force quantities. Geometric interpretation of this approximation is illustrated in Fig. 6. This approximation significantly improves the convergence of the solution.

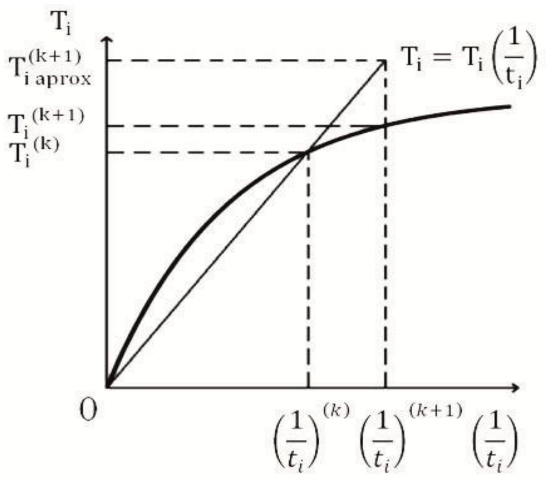

Fig. 3. Principle of linear approximation of force quantities
Rys. 3. Zasada aproksymacji liniowej wielkości sił

Procedure of predicted stress calculation in (k+1)-th step

For each of the possible design variable it is necessary to make an estimation of the force quantities expressed by equations (1) and (2). Approximated values of forces and moments for the n-th design variable from the i-th optimizing group in the (k+1)-th iteration step will be

$$F_{xx(i,n)}^{(k+1)} = \frac{F_{xx(i)}^{(k)} t_i^{(k)}}{t_{(n)}},\ F_{yy(i,n)}^{(k+1)} = \frac{F_{yy(i)}^{(k)} t_i^{(k)}}{t_{(n)}},\quad F_{xy(i,n)}^{(k+1)} = \frac{F_{xy(i)}^{(k)} t_i^{(k)}}{t_{(n)}} \tag{8}$$

$$M_{xx(i,n)}^{(k+1)} = \frac{M_{xx(i)}^{(k)} t_i^{(k)}}{t_{(n)}},\quad M_{yy(i,n)}^{(k+1)} = \frac{M_{yy(i)}^{(k)} t_i^{(k)}}{t_{(n)}},\qquad M_{xy(i,n)}^{(k+1)} = \frac{M_{xy(i)}^{(k)} t_i^{(k)}}{t_{(n)}} \tag{9}$$

Consequently the stress components will be calculated with using approximated values of the force quantities, i.e.
- membrane stress components

$$\sigma_{m_{xx(i,n)}}^{(k+1)} = \frac{F_{xx(i,n)}^{(k+1)}}{t_{(n)}},\quad \sigma_{m_{yy(i,n)}}^{(k+1)} = \frac{F_{yy(i,n)}^{(k+1)}}{t_{(n)}},\quad \sigma_{m_{xy(i,n)}}^{(k+1)} = \frac{F_{xy(i,n)}^{(k+1)}}{t_{(n)}} \tag{10}$$

- bending stress components

$$\sigma_{b_{xx(i,n)}}^{(k+1)} = \frac{6M_{xx(i,n)}^{(k+1)}}{\left(t_{(n)}\right)^2},\quad \sigma_{b_{yy(i,n)}}^{(k+1)} = \frac{6M_{yy(i,n)}^{(k+1)}}{\left(t_{(n)}\right)^2},\quad \sigma_{b_{xy(i,n)}}^{(k+1)} = \frac{6M_{xy(i,n)}^{(k+1)}}{\left(t_{(n)}\right)^2} \tag{11}$$

Then the von Misesstresses on top and bottom surface of the shell are calculated as follows
- equivalentstress on top surface

$$\sigma_{h_{(i,n)}}^{(k+1)} = \sqrt[2]{\begin{gathered}\left(\sigma_{m_{xx(i,n)}}^{(k+1)} + \sigma_{b_{xx(i,n)}}^{(k+1)}\right)^2 + \left(\sigma_{m_{yy(i,n)}}^{(k+1)} + \sigma_{b_{yy(i,n)}}^{(k+1)}\right)^2 - \left(\sigma_{m_{xx(i,n)}}^{(k+1)} + \sigma_{b_{xx(i,n)}}^{(k+1)}\right)\left(\sigma_{m_{yy(i,n)}}^{(k+1)} + \sigma_{b_{yy(i,n)}}^{(k+1)}\right) + \\ +3\left(\sigma_{m_{xy(i,n)}}^{(k+1)} + \sigma_{b_{xy(i,n)}}^{(k+1)}\right)^2\end{gathered}} \tag{12}$$

- equivalentstress on bottom surface

$$\sigma_{d_{(i,n)}}^{(k+1)} = \sqrt[2]{\begin{gathered}\left(\sigma_{m_{xx(i,n)}}^{(k+1)} - \sigma_{b_{xx(i,n)}}^{(k+1)}\right)^2 + \left(\sigma_{m_{yy(i,n)}}^{(k+1)} - \sigma_{b_{yy(i,n)}}^{(k+1)}\right)^2 - \left(\sigma_{m_{xx(i,n)}}^{(k+1)} - \sigma_{b_{xx(i,n)}}^{(k+1)}\right)\left(\sigma_{m_{yy(i,n)}}^{(k+1)} - \sigma_{b_{yy(i,n)}}^{(k+1)}\right) + \\ +3\left(\sigma_{m_{xy(i,n)}}^{(k+1)} - \sigma_{b_{xy(i,n)}}^{(k+1)}\right)^2\end{gathered}} \tag{13}$$

Consequently the stresses in top and bottom surface are compared and one of them is set as maximum ${\sigma_{max}}_{(i,n)}^{(k+1)}$.

3. COMPARISON STUDY OF THE PROPOSED ALGORITHMS

Presented computational algorithms were tested and compared on the structure shown in Fig. 4. Four-node thin shell isoparametric finite elements were used. The pipe with diameter of 80 mm, length of 1000 mm and wall thickness of 10 mm was modelled by means of beam elements. The pipe was connected with the shell by truss elements which transfer only compression forces (nonlinear model with very low tension stiffness). Linear elastic isotropic material model with Young's modulus E=2,1.10^{5}MPa and Poisson's ratio ϑ=0.3 was used. Boundary conditions were defined as follows

- forces =>uniformly distributed force F=10 N.mm^{-1} on the length of the pipe in direction of Y= -1, Z= -1 (Fig. 4, magenta part),
- displacements => zero displacement on the edge in Y-axis direction, zero displacements in all direction on the annular areas under the screws, zero displacement in X-axis direction and zero rotations about Y and Z-axis (symmetry) on the free end of the pipe (Fig. 4, green part).

Three optimizing variables were selected for the process of optimization (see Fig. 5-red, green and blue parts). Maximum design stress was considered as σ_L=*120* MPa. Vector $\mathbf{t}_{start}$=[*40,40,40*] mm was suggestedas the start point and discrete design variables were chosen from the interval of<8, 40> mm with increment of1 mm.Maximum number of iterations was set to 20. Optimizing process was terminated when the following convergence conditions were fulfilled

- stress convergence condition $\left|\frac{\sigma_L-\sigma_i}{\sigma_L}\right| \leq 0.1\,; \qquad \sigma_L-\sigma_i>0,$
- design variable convergence condition $\frac{t_i^{(k)}-t_i^{(k+1)}}{t_i^{(k)}} \leq 0.05\,; \qquad \sigma_L-\sigma_i>0.$

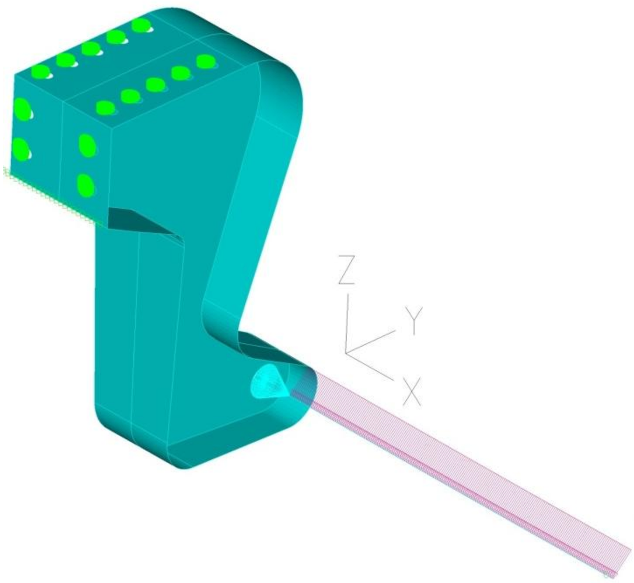

Fig. 4. Boundary conditions for the testing structure
Rys 4. Skrajne warunki dla testowanej konstrukcji

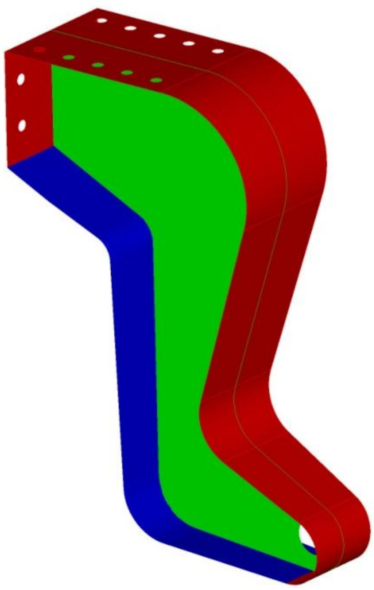

Fig. 5. Optimizing groups (1 – red, 2 – green, 3 – blue)
Rys. 5. Grupy optymalizacyjne (1. – czerwona, 2. – zielona, 3. – niebieska)

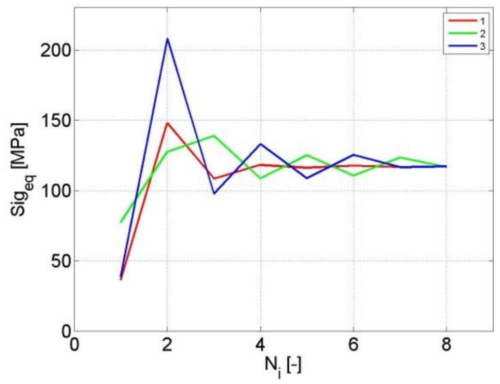

Fig. 6. History of convergence for the algorithm no.1: max. equivalent stress vs. iteration number

Rys. 6. Przebieg konwergencji dla algorytmu nr 1: zależność maksymalnego naprężenia od liczby kroków iteracji

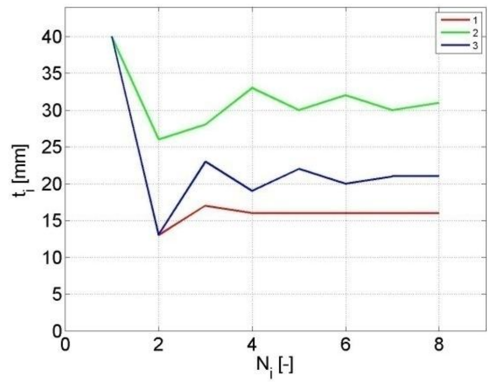

Fig. 7. History of convergence for the algorithm no.1: thickness vs. iteration number

Rys. 7. Przebieg konwergencji dla algorytmu nr 1: zależność grubości od liczby kroków iteracji

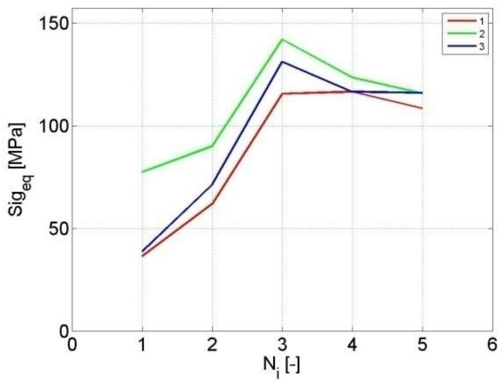

Fig. 8. History of convergence for the algorithm no.2: max. equivalent stress vs. iteration number

Rys. 8. Przebieg konwergencji dla algorytmu nr 2: zależność maksymalnego naprężenia od liczby kroków iteracji

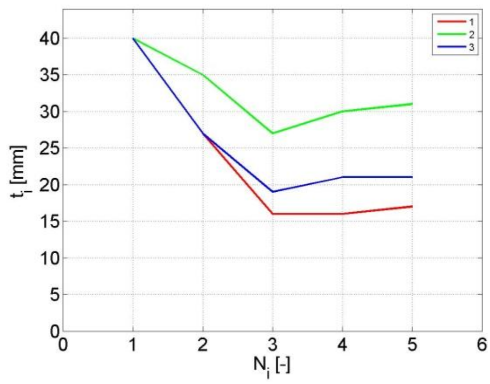

Fig. 9. History of convergence for the algorithm no.2: thickness vs. iteration number

Rys. 9. Przebieg konwergencji dla algorytmu nr 2: zależność grubości od liczby kroków iteracji

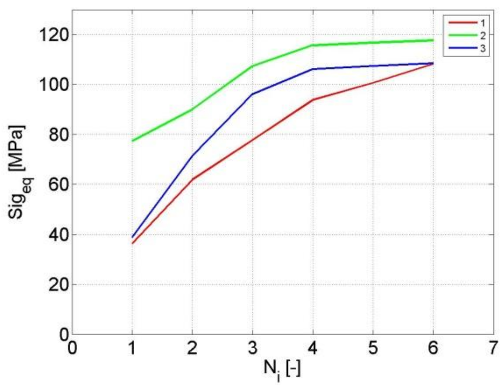

Fig. 10. History of convergence for the algorithm no.3: max. equivalent stress vs. iteration number

Rys. 10. Przebieg konwergencji dla algorytmu nr 3: zależność maksymalnego naprężenia od liczby kroków iteracji

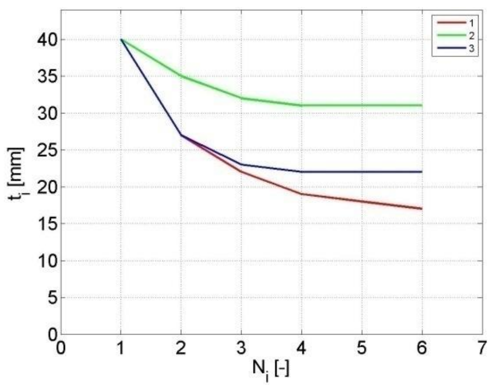

Fig. 11. History of convergence for the algorithm no.3: thickness vs. iteration number

Rys.11. Przebieg konwergencji dla algorytmu nr 3: zależność maksymalnego naprężenia od liczby kroków iteracji

4. CONCLUSION

Our paper deals with theoretical principles and numerical realization of the three fullystressing optimizing algorithms focusing on shell finite elements. The original computational procedures were inbuilt into MATLAB's software module MAT_FSD which cooperates with FE software ADINA. The presented results of the study and authors experience mention the fact that using of classical fully stress design method for shell structures modelled by finite element method can be inconvenient. The authors proposed two new algorithms which have solved this fact. These methods converge well, they are effective in the number of iteration steps and they have big perspective for large optimizing problems where the goal is to find hundreds of structural parameters by application of relatively low number of iteration steps.

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